

203 81413 #9 4, 10, 15-20, 33-38, 48, 51, 53, 77

(4) h = height of wave as function
of v = velocity of wind km/h t = duration
hours

$h(v, t)$

(a) $\frac{\partial h}{\partial v}$ = change in height over a small
increment of velocity.

$\frac{\partial h}{\partial t}$ = change in height over small increment
of time.

(b) we estimate $f_v(40, 15)$ and $f_t(40, 15)$

$$f(40, 15) = 25$$

$$f(50, 15) = 36$$

$$f(30, 15) = 16$$

$$\frac{f(50, 15) - f(40, 15)}{50 - 40} = \frac{36 - 25}{10} = \frac{11}{10} = 1.1$$

$$\frac{f(40, 15) - f(30, 15)}{40 - 30} = \frac{25 - 16}{10} = \frac{9}{10}$$

AVG = $1 \frac{\text{ft}}{\text{km/h}}$

$$f_v(40, 15) \approx 1 \frac{\text{ft}}{\text{km/h}}$$

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(4) (b) ent'd: $f_t(40, 15)$: v is fixed @ $v = 40$.

$$f(40, 15) = 25, \quad f(40, 20) = 28$$

$$f(40, 10) = 21$$

$$\frac{f(40, 20) - f(40, 15)}{20 - 15} = \frac{28 - 25}{5} = \frac{3}{5} \frac{\text{ft}}{\text{hr}}$$
$$\frac{f(40, 15) - f(40, 10)}{15 - 10} = \frac{25 - 21}{5} = \frac{4}{5} \frac{\text{ft}}{\text{hr}}$$

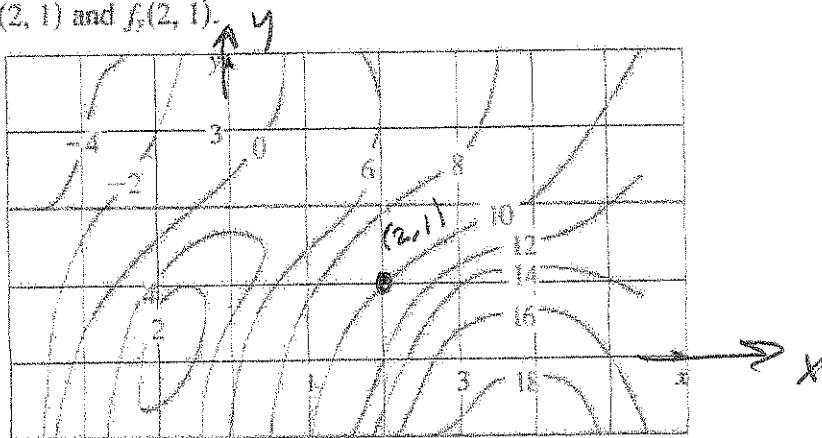
} AVG = $\frac{3+4}{10} = \frac{7}{10}$

$f_t(40, 15) \approx \frac{7}{10} \frac{\text{ft}}{\text{hr}}$ \approx rate at which wave heights are rising wrt time, with wind velocity held constant @ 40 knots.

(c) $\lim_{t \rightarrow \infty} \frac{dh}{dt} = 0$, as wave heights seem to stabilize out past 30 or 40 hrs.

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16. A contour map is given for a function f . Use it to estimate $f_x(2, 1)$ and $f_y(2, 1)$.



$$f_x(2, 1) : \begin{aligned} f(2, 1) &= 10 \\ f(3, 1) &= 14 \\ f(1, 1) &= 8 \end{aligned}$$

$$\frac{f(2, 1) - f(3, 1)}{2 - 3} = \frac{10 - 14}{-1} = 4$$

$$\frac{f(2, 1) - f(1, 1)}{2 - 1} \approx \frac{10 - 8}{1} = 2$$

$$\text{Avg} \uparrow f_x(2, 1) \approx 3.$$

$$f_y(2, 1) : \begin{aligned} f(2, 1) &= 10 \\ f(2, 2) &\approx 8 \\ f(2, 0) &= 12 \end{aligned}$$

$$\frac{f(2, 1) - f(2, 2)}{1 - 2} = \frac{10 - 8}{-1} = -2$$

$$\frac{f(2, 1) - f(2, 0)}{1 - 0} = \frac{10 - 12}{1} = -2$$

$$\text{Avg} \uparrow f_y(2, 1) \approx -2$$

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#5 15-40 Find 1st partials

$$(5) f(x, y) = y^5 - 3xy$$

$$f_x = -3y$$

$$f_y = 5y^4 - 3x$$

$$(16) f(x, y) = x^4 y^3 + 8x^2 y$$

$$f_x = 4x^3 y^3 + 16xy$$

$$f_y = 3x^4 y^2 + 8x^2$$

$$(17) f(x, t) = e^{-t} \cos(\pi x)$$

$$f_x = -\pi e^{-t} \sin(\pi x)$$

$$f_t = -e^{-t} \cos(\pi x)$$

$$(18) f(x, t) = \sqrt{x} \ln t$$

$$f_x = \frac{1}{2} x^{-\frac{1}{2}} \ln t$$

$$f_t = \frac{\sqrt{x}}{t}$$

203 § 14.3 #s 19, 20, 33-38, 48, 51, 53, 77

(19) $z = (2x + 3y)^{10}$

$$z_x = 10(2x + 3y)^9 (2)$$

$$z_y = 10(2x + 3y)^9 (3)$$

(20) $z = \tan(xy)$

$$z_x = y \sec^2(xy)$$

$$z_y = x \sec^2(xy)$$

(33) $w = \ln(x + 2y + 3z)$

$$w_x = \frac{1}{x + 2y + 3z}, \quad w_y = \frac{2}{x + 2y + 3z}, \quad w_z = \frac{3}{x + 2y + 3z}$$

(34) $w = z e^{xy}$

$$w_x = y z^2 e^{xy}, \quad w_y = x z^2 e^{xy}, \quad w_z = xy z e^{xy}$$

(35) $u = xy \sin^{-1}(yz)$

$$u_x = y \sin^{-1}(yz) + \cancel{\left(xy \frac{1}{\sqrt{1-(yz)^2}} \right)} (0)$$

$$u_y = x \sin^{-1}(yz) + \left(xy \frac{1}{\sqrt{1-(yz)^2}} \right) z$$

$$u_z = \left(xy \frac{1}{\sqrt{1-(yz)^2}} \right) y$$

$$\frac{d}{dx} [f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$$

$$= \frac{1}{\cos(\sin^{-1}(x))}$$



203 JN, 3 #s 36-38, 40, 51, 53, 77

(36) $u = x^{y/2} = x^{yz^{-1}}$

$$u_x = (yz^{-1}) x^{yz^{-1}-1}$$

$$u_y = \left(\frac{1}{z} \ln x\right) x^{\frac{y}{z}}$$

$$\frac{d}{dx} \ln u = \left(\frac{y}{z}\right) \ln x$$

$$\frac{u'}{u} = \frac{y}{zx}$$

$$\frac{du}{dx} = \frac{y}{zx} x^{yz^{-1}}$$

$$= \frac{y}{z} x^{yz^{-1}-1}$$

$$\frac{d}{dy} : x^{\frac{y}{z}} = e^{\ln(x^{\frac{y}{z}})}$$

$$= e^{\frac{y}{z} \ln x}$$

$$= e^{\left(\frac{\ln x}{z}\right) y} \rightarrow$$

$$\frac{d}{dy} (x^{y/2}) = \frac{\ln x}{z} e^{\frac{\ln x}{z} y}$$

$$= \left(\frac{1}{z} \ln x\right) e^{\frac{y}{z} \ln x}$$

$$= \left(\frac{1}{z} \ln x\right) e^{\ln(x^{\frac{y}{z}})}$$

$$= \left(\frac{1}{z} \ln x\right) x^{\frac{y}{z}}$$

I can never remember

$$\frac{d}{dx} [b^x] = (\ln(b)) b^x$$

So I re-derive it,

any time it's been

a while...

203 §. 14.3 #3 37-0, 48, 51, 53, 77

$$(37) h(x, y, z, t) = x^2 y \cos\left(\frac{z}{t}\right)$$

$$h_x = 2xy \cos\left(\frac{z}{t}\right)$$

$$h_y = x^2 \cos\left(\frac{z}{t}\right)$$

$$h_z = x^2 y \left(-\sin\left(\frac{z}{t}\right)\right) \left(\frac{1}{t}\right) = -\frac{x^2 y}{t} \sin\left(\frac{z}{t}\right)$$

$$h_t = x^2 y \left(-\sin\left(\frac{z}{t}\right)\right) \left(-\frac{z}{t^2}\right) = \frac{x^2 y z}{t^2} \sin\left(\frac{z}{t}\right)$$

$$(38) \phi(x, y, z, t) = \frac{\alpha x + \beta y^2}{\gamma z + \delta t^2}$$

$$\phi_x = \frac{\alpha}{\gamma z + \delta t^2} \left[\frac{1}{\gamma z + \delta t^2} (\alpha x + \beta y^2) \right] = \boxed{\frac{\alpha}{\gamma z + \delta t^2}}$$

$$\phi_y = \frac{\beta}{\gamma z + \delta t^2}$$

$$\phi_z = \frac{\alpha}{\gamma z + \delta t^2} \left[(\alpha x + \beta y^2) (\gamma z + \delta t^2)^{-1} \right]$$

$$= (-1) (\gamma z + \delta t^2)^{-2} (\gamma) (\alpha x + \beta y^2)$$

$$= -\frac{\gamma (\alpha x + \beta y^2)}{(\gamma z + \delta t^2)^2}$$

$$\phi_t = (-1) (\gamma z + \delta t^2)^{-2} (2\delta t) (\alpha x + \beta y^2)$$

$$= -\frac{2\delta t (\alpha x + \beta y^2)}{(\gamma z + \delta t^2)^2}$$

203 §14.3#s 48, 51, 53, 77

~~48~~ #s 47-50 use implicit diff to

$$\text{find } \frac{dz}{dx} \text{ \& } \frac{dz}{dy}$$

(48) $x^2 - y^2 + z^2 - 2z = 4$

$$z_x = \frac{dz}{dx} :$$

$$2x + 2z z_x - 2z_x = 0$$

$$(2z - 2) z_x = -2x$$

$$z_x = \frac{-2x}{2z - 2}$$

$$z_y = \frac{dz}{dy} :$$

$$-2y + 2z z_y - 2z_y = 0$$

$$(2z - 2)(z_y) = 2y$$

$$z_y = \frac{2y}{2z - 2}$$

See Formula ;
pg 954, §14.5.

I usually don't
bother write it, but
you might like
using it.

Easier for me to just use the
old skills. Technically, we
don't "know" formula 7, yet.

203 § 14.3 #5 51, 53, 77

#5 51-2 Find z_x & z_y

(51) $z = f(x) + g(y)$

$$z_x = f'(x), \quad z_y = g'(y)$$

(b) $z = f(x+y) \rightarrow$

$$z_x = f'(x+y)$$

$$z_y = f'(x+y)$$

(53) Find all 2nd partials

$$f(x,y) = x^3 y^5 + 2x^4 y$$

$$f_x = 3x^2 y^5 + 8x^3 y$$

$$f_y = 5x^3 y^4 + 2x^4$$

$$f_{yy} = 20x^3 y^3$$

$$f_{xx} = 6xy^5 + 24x^2 y$$

$$f_{xy} = 15x^2 y^4 + 8x^3 = f_{yx} = 15x^2 y^4 + 8x^3 \checkmark$$

(77) Verify: $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ is solution of Laplace's eq'n in 3-D: $u_{xx} + u_{yy} + u_{zz} = 0$

$$u_x = -\frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{3}{2}}(2x)$$

$$u_y = -\frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{3}{2}}(2y)$$

$$u_z = -\frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{3}{2}}(2z)$$

203 § 14.3 #77

$$u_{xx} = \frac{3}{4}(x^2+y^2+z^2)^{-\frac{5}{2}}(2x)(2x) - \frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2)$$

$$u_{yy} = \frac{3}{4}(x^2+y^2+z^2)^{-\frac{5}{2}}(2y)(2y) - \frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2)$$

$$u_{zz} = \frac{3}{4}(x^2+y^2+z^2)^{-\frac{5}{2}}(2z)(2z) - \frac{1}{2}(x^2+y^2+z^2)^{-\frac{3}{2}}(2)$$

$$\Rightarrow u_{xx} + u_{yy} + u_{zz}$$

$$= \frac{3x^2 + 3y^2 + 3z^2}{(x^2+y^2+z^2)^{5/2}} - \frac{3}{(x^2+y^2+z^2)^{3/2}}$$

$$= 3 \left[\frac{x^2+y^2+z^2}{(x^2+y^2+z^2)^{5/2}} - \frac{(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}} \right] = 0$$



Times

$$\frac{x^2+y^2+z^2}{x^2+y^2+z^2}$$