203 §14.2 #s 5, 8, 13, 16, 25, 34*

#s 5–22 find limit, if $\exists$, or show it $\nexists$.

(5) $\lim_{(x,y)\to(1,2)} (5x^3 - x^2y^2) = 5(1)^3 - (1)^2(2)^2$

$= 5 - 4 = \boxed{1}$

polynomial $\Longrightarrow$ cnt^s.

(8) $\lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right)$ $\mathcal{D}$: $x \neq 0$

and need $\frac{y^2+1}{x^2+xy} > 0 \longleftrightarrow x(x+y) > 0$

ALL TRUE stuff, but not to the point

$x > 0 \Longrightarrow$ Need $x + y > 0$

$x > -y$

$y > -x$

$\ln\left(\frac{1+0^2}{1^2+1(0)}\right)$

$= \ln(1) = 0$

$x < 0 \Longrightarrow$ Need $x + y < 0$

$x < -y$

$y < -x$

$\frac{1+y^2}{x^2+xy}$ is cnt^s on its domain

$\ln(t)$ " " " "

$(1,0) \in \mathcal{D}\left(\frac{1+y^2}{x^2+xy}\right)$ & $t = \frac{1+0^2}{1+1(0)} = 1 \in \mathcal{D}(\ln(t))$

$\Longrightarrow \lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right) = \ln(1) = \boxed{0}$

203 $S^1$14.2 #5 13, 16, 25, 39 *

(11) $\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{x^4+y^4}$. Not cont @ (0,0) $\notin \mathcal{D}$

$\frac{y^2\sin^2 x}{x^4+y^4}$. Along $y=0$: $\frac{0^2\sin^2 x}{x^4+0^4} = \frac{0}{x^4} = 0$

Along $y=x$: $\frac{x^2\sin^2 x}{2x^4} = \frac{\sin^2 x}{2x^2}$

$\xrightarrow{(x,y)\to(0,0)} \frac{1}{2} \neq 0$, so

two different limits, from 2 different approaches to $(0,0) \Longrightarrow \boxed{\nexists}$

(13) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ $(0,0) \notin \mathcal{D}$

Along $x=0$: $f(x,y) = 0 = \lim f(x,y)$

Along $x=y$: $\frac{x^2}{2x^2} \xrightarrow{x\to 0} \frac{1}{2} = \lim f(x,y)$

$0 \neq \frac{1}{2} \Longrightarrow \lim f(x,y) \nexists$.

(16) $\lim_{(x,y)\to(0,0)} \frac{x^2\sin^2 y}{x^2+2y^2} = \lim f(x,y)$ $(0,0) \notin \mathcal{D}$

Along $x=0$: $f(x,y) = 0$

" $y=x$: $f(x,y) = \frac{x^2\sin^2 x}{3x^2} = \frac{\sin^2 x}{3} \longrightarrow 0$

203 §14.2 #s 16, 25, 39

(16) cntd. Not obvious how to show how it fails. Take a step back.

$$\frac{x^2 \sin^2 y}{x^2+2y^2} \le \sin^2 y, \text{ since } \frac{x^2}{x^2+2y^2} \le 1.$$

and $0 \le f(x,y) \le \sin^2 y$

$\downarrow (x,y)\to 0 \qquad \downarrow (x,y)\to 0$

$0 \qquad\qquad 0$

$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) = 0 \ !}$$

(25) Find $h(x,y) = g(f(x,y))$ and the set on which it's cont.

$g(t) = t^2 + \sqrt{t}$, $f(x,y) = 2x+3y-6$

$\mathcal{D}(g(f)) = \{(x,y) \mid (x,y) \in \mathcal{D}(f) \text{ and } f(x,y) \in \mathcal{D}(g)\}$

$\mathcal{D}(f) = \mathbb{R}\times\mathbb{R} = \{(x,y) \mid x,y \in \mathbb{R}\}$

$\mathcal{D}(g) = \{t \mid t \ge 0\}$ Need $f(x,y) \ge 0$;

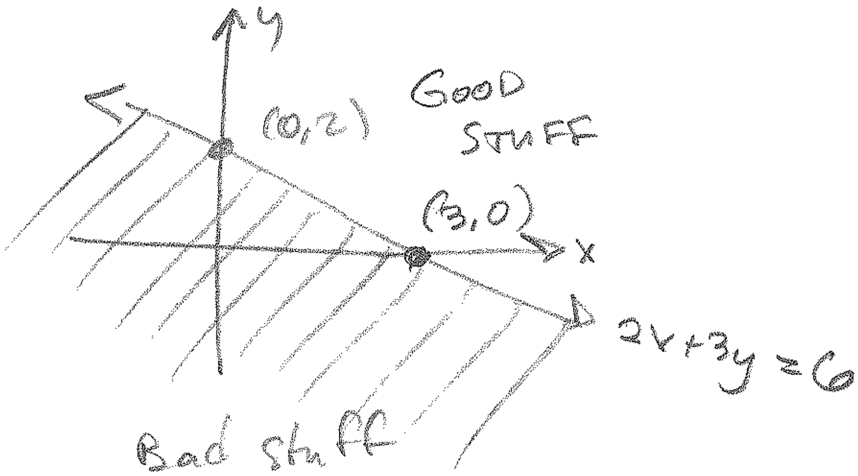
203 §14.2 #s 25, 39

25 cnt'd. Need $f(x,y) = 2x+3y-6 \geq 0$

$\Rightarrow 3y \geq -2x + 6$

$y \geq -\frac{2}{3}x + 2$

$\therefore$ h's domain is $\{(x,y) \mid y \geq -\frac{2}{3}x + 2\}$



39 Use polar coords to find the limit.

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2} \qquad x = r\cos\theta \quad y = r\sin\theta$$

$$\frac{r^3\cos^3\theta + r^3\sin^3\theta}{r^2\cos^2\theta + r^2\sin^2\theta} = r\cos^3\theta + r\sin^3\theta$$

Book says: $r \geq 0 \Rightarrow (x,y) \to (0,0)$ means $r \to 0^+$

$$\xrightarrow{r \to 0^+} \boxed{0 = \lim f(x,y)}$$