203 §14.1 #s 5, 7, 10, 11, 16, 17, 20, 21, 23,
27, 29-31, 43, 49, 50, 65, 67, 68

⑤ $S = f(w, h) = .1091 w^{.425} h^{.725}$ models surface area as function of height & weight,
inches pounds

ⓐ $f(160, 70) = .1091(160)^{.425}(70) \approx 20.5244639\ ft^2$
is surface area for person 5'10" & 160 lbs

ⓑ My own S.A. Bleah

⑦ Wave height h, is modeled by
$h(v, t) =$ TABLE pg 912, where $v =$ wind velocity in knots & $t =$ time in hours. h is in units of feet.

ⓐ. $h(40, 15) = 5$ ft says wave ht is 5 ft when wind has been blowing at 40 knots for 15 hours.

ⓑ $h(30, t)$ is height as func of t, with wind speed fixed at 30 knots. It's an increasing function of time.

ⓒ $h(v, 30)$ is $h(v)$ with time fixed @ $t = 30$ hrs.
It's an increasing function (Read down 30 column)

203 §14.1 #s 10, 11, 16, 17, 20, 21, 23

(10) $F(x,y) = 1 + \sqrt{4-y^2} \longrightarrow$

(a) $\boxed{F(3,1) = 1 + \sqrt{3}}$

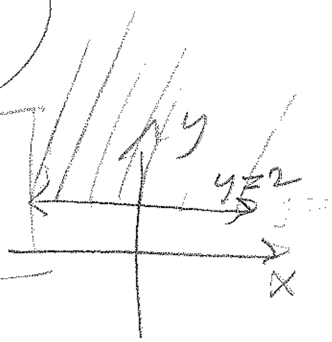
(b) $\mathcal{D}(F)$: Need $4-y^2 \geq 0$

$y^2 \leq 4$ $\quad \overset{+ \quad\quad - \quad\quad +}{\longleftrightarrow}$ $\quad \leq 0$

$\quad\quad\quad\quad\quad -2 \quad\quad 2$

$(y-2)(y+2) \leq 0$

$\longrightarrow \boxed{\mathcal{D}(F) = \{(x,y) \mid |y| \leq 2, x \in \mathbb{R}\}}$



Bad stuff shaded

(c) $R(f) = R\left(1 + \sqrt{4-y^2}\right)$

This is a cylinder.

$z = 1 + \sqrt{4-y^2}$

$(z-1)^2 = 4 - y^2$

$y^2 + (z-1)^2 = 4$

Top half of a circle of radius 2 centered at $(x,0,1)$ as cross section @ $x$.

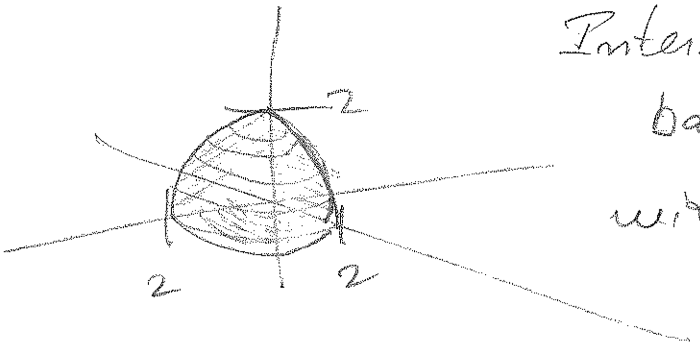
$\boxed{R = [0,2]}$

203 §14.1 #s 11, 16, 17, 20, 21, 23

(11) $f(x,y,z) = \sqrt{x}+\sqrt{y}+\sqrt{z}+\ln(4-x^2-y^2-z^2)$

~~R~~ (a) $f(1,1,1) = 1+1+1+\ln(1)$ $\boxed{=3}$

(b) $D$: $x \ge 0, y \ge 0, z \ge 0, x^2+y^2+z^2 \le 4$



Intersection of the ball $x^2+y^2+z^2 \le 4$ with 1st octant.

Basically $\frac{1}{8}$ of ball.

(c) $R$: $x=y=z=0$, $\ln(4)$

Ugh!
Good thing they didn't ask that!

#s 13-22 Find and sketch $D$

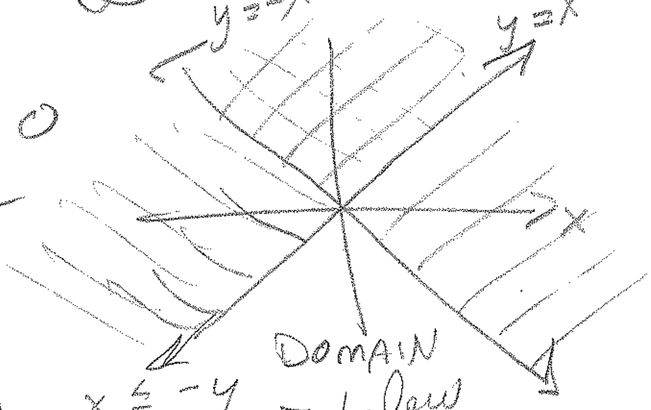
(16) $f(x,y)=\sqrt{x^2-y^2}$

$x^2-y^2 \ge 0$

$x^2 \ge y^2$

$|x| \ge y$

$x \ge y$ OR $x \le -y$



DOMAIN is below the 2 lines

203 §14.1 #s ~~16~~, 17, 20, 21, 23

(17) $f(x,y)=\sqrt{1-x^2}-\sqrt{1-y^2}$

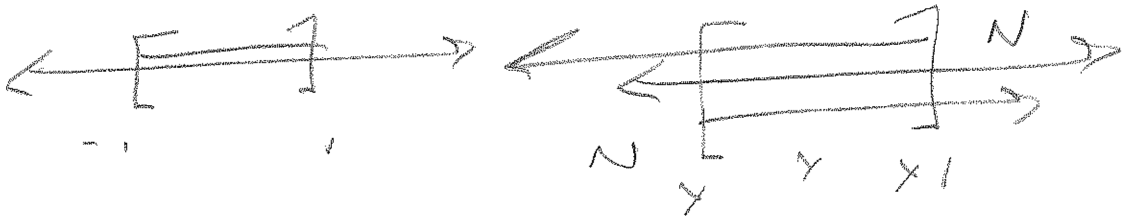
Need $1-x^2 \geq 0$ and $1-y^2 \geq 0$

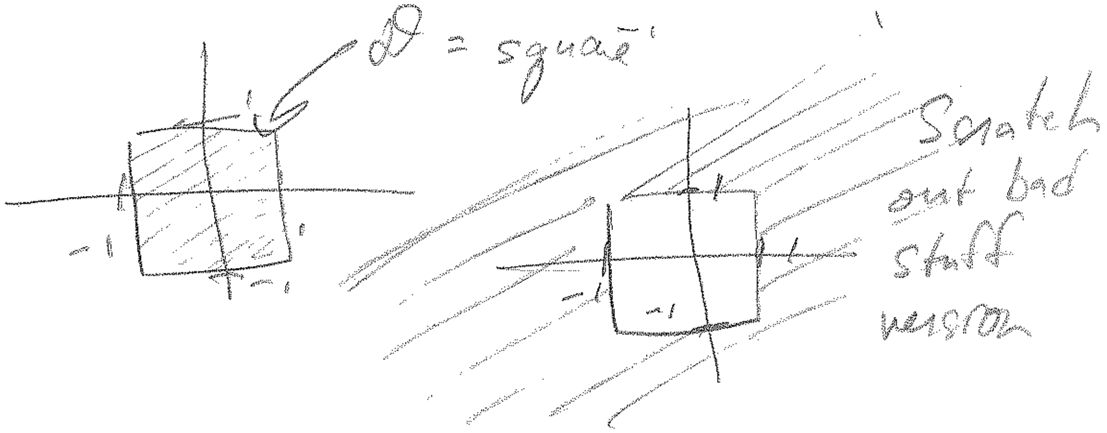
$1 \geq x^2$ $\qquad$ $1 \geq y^2$

$x^2 \leq 1$ $\qquad$ $y^2 \leq 1$

$|x| \leq 1$ $\qquad$ $|y| \leq 1$

$x \leq 1$ and $x \geq -1$ $\qquad$ $y \leq 1$ and $y \geq -1$ AND





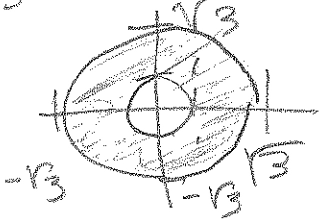
(20) $f(x,y) = \arcsin(x^2+y^2-2)$

$\mathcal{D}$ Need $|x^2+y^2-2| \leq 1$

$x^2+y^2-2 \leq 1$ and $x^2+y^2-2 \geq -1$

$x^2+y^2 \leq 3$ and $x^2+y^2 \geq 1$

In



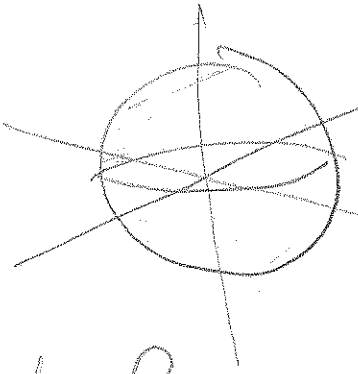
Shaded region is $\mathcal{D}$

203 §14.1 #s 21, 23, 27, 29-31, 49, 50, 65, 67, 68

(21) $f(x,y,z) = \sqrt{1-x^2-y^2-z^2}$

Need $1-x^2-y^2-z^2 \geq 0$, ⇒

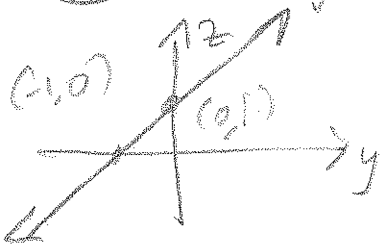
$x^2+y^2+z^2 \leq 1$ → unit ball, centered at the origin



Solid sphere = Ball.

#s 23-31 Sketch f,

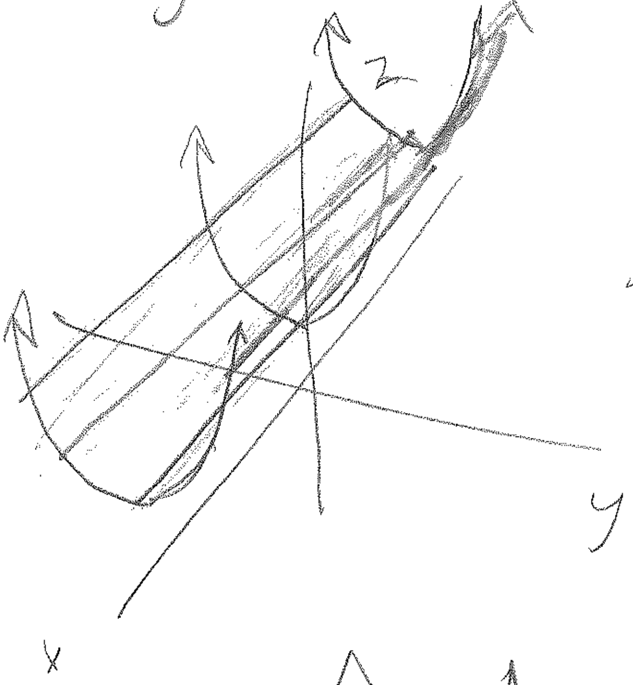
(23) $f(x,y) = 1+y$



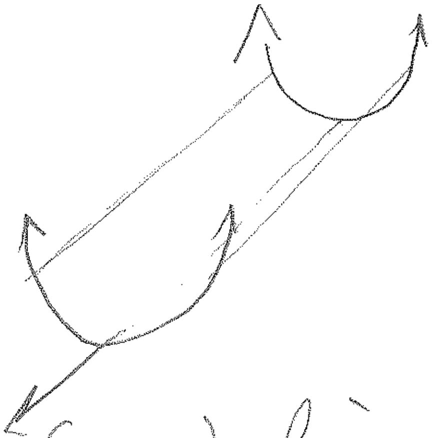
Extend this line in x-direction, will give a plane

203 §14.1 #s 27, 29-31, 49, 50, 65, 67, 68

㉗ $f(x,y) = y^2+1$ Another cylinder with rulings parallel to x-axis



Trough with parabolic cross-sections, with long axis parallel to x-axis



(x, 0, 1) line is bottom of trough

203 §14.1 #s 29-31, 49, 50, 65, 67, 68

(29) $f(x,y) = 9 - x^2 - 9y^2$

$z=0$: $9y^2+x^2=9$

$x^2+9y^2=9$

$\frac{x^2}{9}+y^2=1$

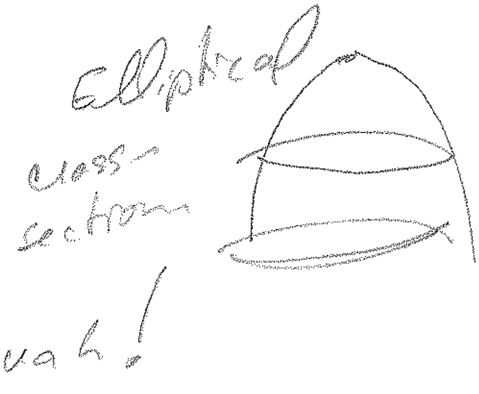
$z=1$: $9-x^2-9y^2=1$

$x^2+9y^2=8$

$\frac{x^2}{8}+\frac{y^2}{\frac{8}{9}}=1$ Smaller ellipse.

$z=9$: $-x^2-9y^2=0$ The point $(0,0,9)$

$x^2+9y^2=0$



$z>9$: $9-x^2-9y^2=10$

$-x^2-9y^2=1$

$x^2+y^2=-1$ Nevah!

203 §14.1 #s 29-31, 49, 50, 65, 67, 68

$z$ = negative values

$9 - x^2 - 9y^2 = z$

$-x^2 - 9y^2 = z - 9$

$x^2 + 9y^2 = 9 - z$

Ellipses growing as $z$ decreases

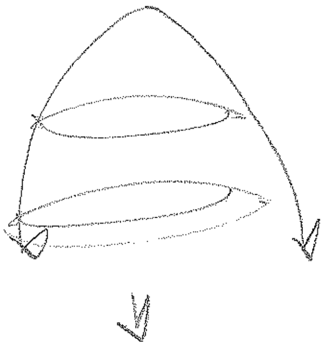
↳ cross-sections

$x = c$ : $9 - x^2 - 9y^2 = z$

parabola in $y$ for slices parallel to $x=0$ plane.

Likewise for $y$, only @ right angles

So, we have an inverted elliptical paraboloid.



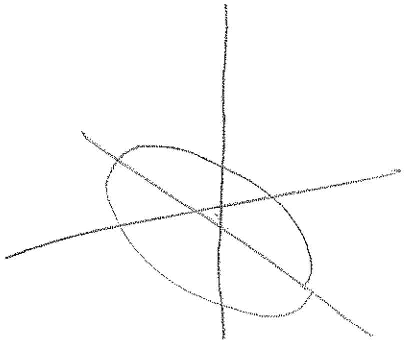
203 §14.1 #s 30,31, 49,50,65,67,68

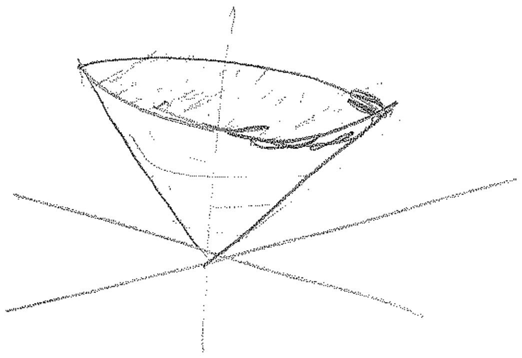
(30) $f(x,y) = \sqrt{4x^2+y^2} = z$

$\Rightarrow z^2 = 4x^2+y^2$ is Cone.

$z = \sqrt{4x^2+y^2}$ is top half.

Contours
$z = c$ are ellipses. Long axis in y-direction





(31) $f(x,y) = \sqrt{4-4x^2-y^2}$

$4-4x^2-y^2 \geq 0$

$4x^2+y^2 \leq 4$

$x^2 + \frac{y^2}{4} \leq 1$

gives domain as interior of ellipse.

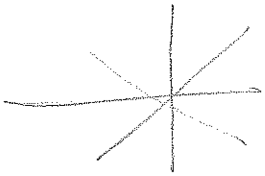
203 §14.1 #s 49,50, 65,67,68

㊾ & ㊿ Draw contour map.

㊾ $f(x,y) = \sqrt{y^2-x^2}$

$z=0$: $y^2-x^2=0 \Rightarrow$

$y=\pm x$



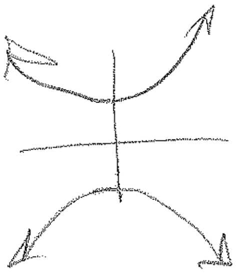
$z=1$: $\sqrt{y^2-x^2}=1$

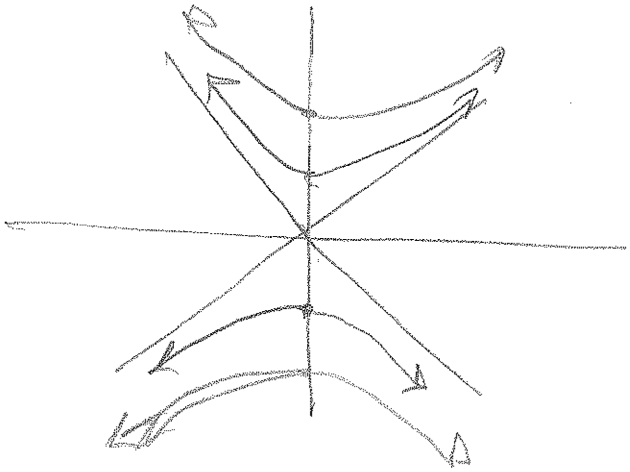
$y^2=x^2+1 \longrightarrow y^2-x^2=1^2$

$y=\pm\sqrt{x^2+1}$

$z=c>0$: $y^2-x^2=c^2$

$\frac{y^2}{c^2}-\frac{x^2}{c^2}=1$





203 §14.1 #s 50, 65, 67, 68

㊿ $f(x,y)=\frac{y}{x^2+y^2}$

$z=0$ : $y=0$, $x=$ any : $x$-axis

$z=c$ : $\frac{y}{x^2+y^2}=c$

$cx^2+cy^2=y$

$cx^2+cy^2-y=0$

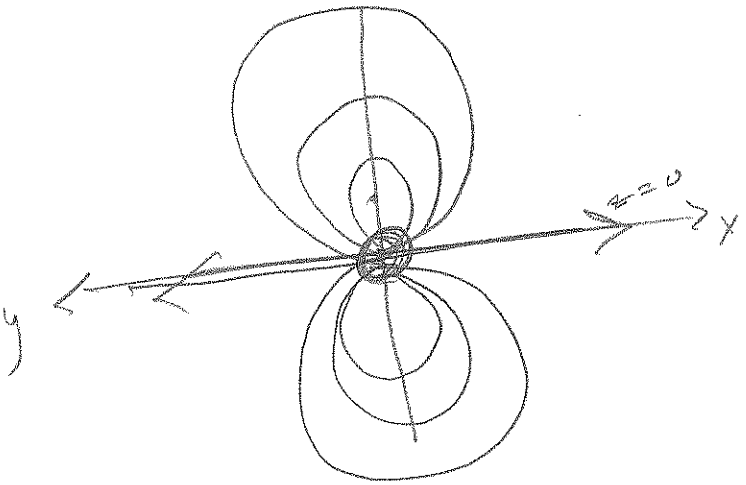
$cx^2+c\left(y^2-\frac{y}{c}+\left(\frac{1}{2c}\right)^2\right)=c\left(\frac{1}{4c^2}\right)=\frac{1}{4c}$

$cx^2+c\left(y-\frac{1}{2c}\right)^2=\frac{1}{4c}$

$4c^2x^2+4c^2\left(y-\frac{1}{2c}\right)^2=1$

$\frac{x^2}{\frac{1}{4c^2}}+\frac{\left(y-\frac{1}{2c}\right)^2}{\frac{1}{4c^2}}=1$

Circles, centered @ $\left(0,\frac{1}{2c}\right)$, radius $r=\frac{1}{2c}$



None include the origin

Like 2 megas touching.

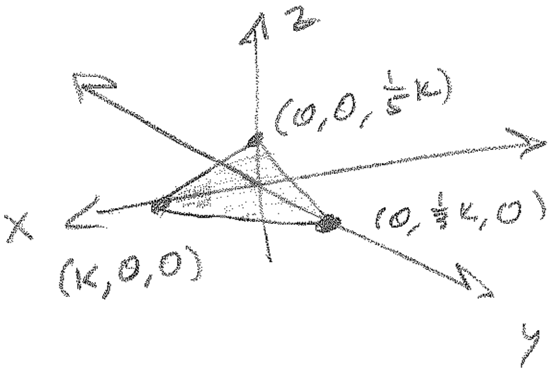
203 §14.1 #s 65, 67, 68

#s 65-68 Describe the level curves

(65) $f(x,y,z) = x + 3y + 5z$

$x + 3y + 5z = K$

$(K,0,0)$, $(0, \frac{1}{3}K, 0)$, $(0, 0, \frac{1}{5}K)$



Planes parallel to
$x + 3y + 5z = 0$, or,
planes ⊥ to $\langle 1,3,5 \rangle$.

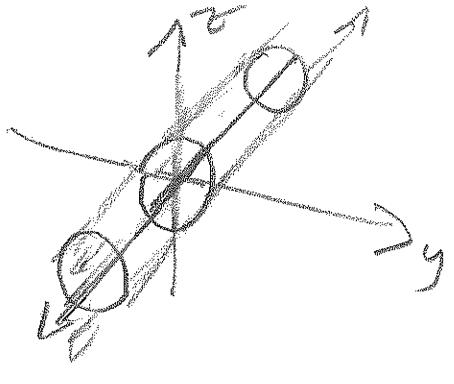
(67) $f(x,y,z) = y^2 + z^2$

$y^2 + z^2 = K$ These are circles, which become right circular cylinders, with long axis (& insides) parallel to x-axis, when x is included

$r = \sqrt{K}$

$K \geq 0$

$K < 0 \longrightarrow \times$



203 §14.1 #68

(68) Describe how graph of $g$ is obtained from the graph of $f$.

(a) $g(x,y) = f(x,y)+2$ $\boxed{\text{up } 2}$

(b) $g(x,y) = 2f(x,y)$ Double $z$-values. Twice as tall, twice as steep.

(c) $g(x,y) = 2 - f(x,y)$

① $-f(x,y)$ Flip upside down

② $-f(x,y)+2$ move up 2.