203 § 13.4 #s 2, 5, 10, 14, 15, 19, 26, 46*

$$\kappa = \frac{|\bar{T}'|}{|\bar{r}'|} = \frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3}$$

$$\bar{a} = \text{acceleration} = v'\,\bar{T} + \kappa v^2 \bar{N}$$
$$= \underline{a_T\,\bar{T} + a_N \bar{N}}$$

↳ Breaking acceleration into two orthogonal components

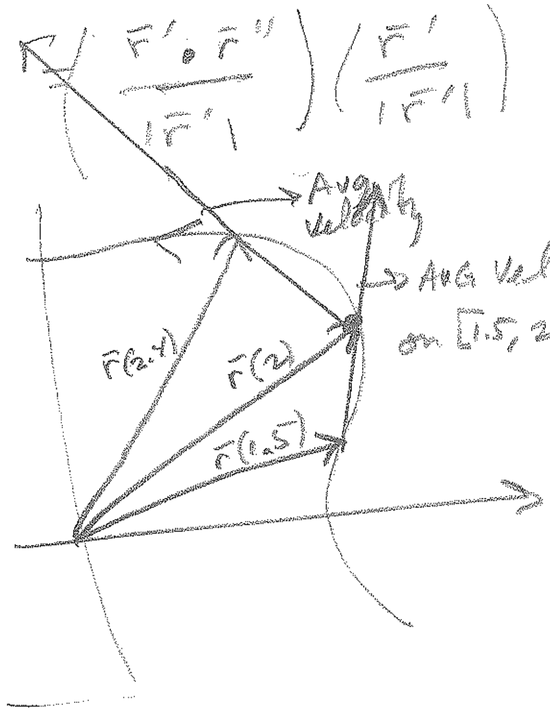
$$a_T = \frac{\bar{r}' \cdot \bar{r}''}{|\bar{r}'|}$$

$$a_N = \kappa v^2 = \frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3} |\bar{r}'|^2 = \frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|}$$

$$\bar{a} = a_T \bar{T} + a_N \bar{N}$$

$$\left(\frac{\bar{r}' \cdot \bar{r}''}{|\bar{r}'|}\right)\left(\frac{\bar{r}'}{|\bar{r}'|}\right) + \left(\frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|}\right) \frac{\bar{T}'}{|\bar{T}'|}$$

(2)



(2) Draw avg velocity $2 \le t \le 2.4$

$$\frac{\bar{r}(2.4) - \bar{r}(2)}{.4} = \frac{5}{2}\left(\bar{r}(2.4) - \bar{r}(2)\right)$$

so $2\frac{1}{2}$ times the length of the vector $\bar{r}(2.4) - \bar{r}(2)$, in that direction

203 §13.4 #5 ~~2~~5, 10 ~~#~~, 15, 19, 26, 46

(a) and (b) Same as (a) but $1.5 \le t \le 2$

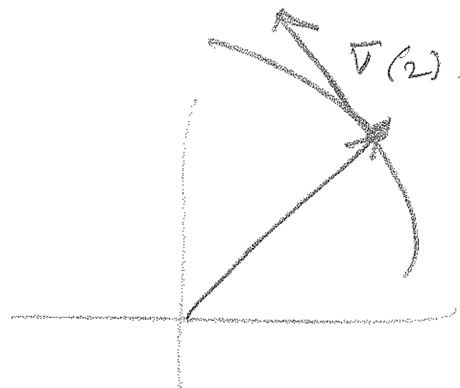
Basically, connect the dots between $\bar{r}(2)$ & $\bar{r}(1.5)$.

Then double the length, since $\dfrac{\bar{r}(2)-\bar{r}(1.5)}{.5}$

$= 2(\bar{r}(2)-\bar{r}(1.5))$

(c) $\bar{v}'(2) = \lim_{h\to 0} \frac{1}{h}\left[\bar{r}(2+h)-\bar{r}(2)\right]$

(d) $\bar{v}'(2)$ on pic



#5 3-8 Find $\bar{v}$, $\bar{a}$, v. Show w/ sketch what these look like. Draw $\bar{v}$ & $\bar{a}$

WE'D USE "s" for "speed," but s is arc length

(5) $r(t) = \langle 3\cos t, 2\sin t\rangle$ (a) $t = \frac{\pi}{3}$

$\boxed{\bar{r}\left(\frac{\pi}{3}\right) = \langle \frac{3}{2}, \cancel{\frac{2\sqrt{3}}{2}} \rangle}$

$= \langle \frac{3}{2}, \sqrt{3}\rangle$

$\bar{r}'(t) = \langle -3\sin t, 2\cos t\rangle$

$\bar{a} = \bar{r}'' = \langle -3\cos t, -2\sin t\rangle$

$\bar{r}'\left(\frac{\pi}{3}\right) = \langle -3\cdot\frac{\sqrt{3}}{2}, 2\cdot\frac{1}{2}\rangle = \boxed{\langle \frac{-3\sqrt{3}}{2}, 1\rangle = \bar{r}'\left(\frac{\pi}{3}\right) = \bar{v}}$

203 §13.4 #9 5, 10, 14, 15, 19, 26 - tk

NOTE: You guys NEED to put bars over vectors!

$\bar{v} = \bar{r}'$ but $v = |\bar{r}'|$. Different deal!

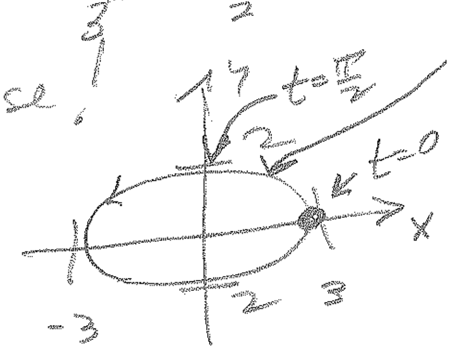
I'm taking off points for failing to use notation that CLEARLY sets vectors apart from scalars.

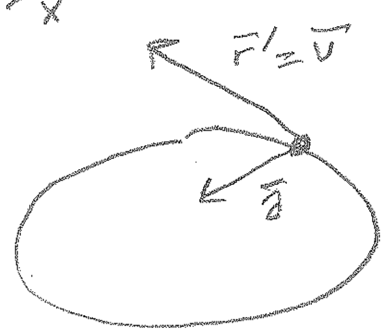
(5) cnt'd $\langle 3\cos t, 2\sin t \rangle$

$$\frac{3\cos t}{3} + \frac{2\sin t}{2} = \frac{x}{3} + \frac{y}{2}, \ \phi$$

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{2}\right)^2 = \frac{x^2}{3^2} + \frac{y^2}{2^2} = \cos^2 t + \sin^2 t = 1,$$

i.e., Ellipse!





$\bar{a} = \bar{r}''\left(\frac{\pi}{3}\right)$

$= \langle -3\cos\frac{\pi}{3}, -2\sin\frac{\pi}{3} \rangle$

$= \boxed{\langle -\frac{3}{2}, -\sqrt{3} \rangle = \bar{a}\left(\frac{\pi}{3}\right)}$

$v\left(\frac{\pi}{3}\right) = \left|\bar{v}\left(\frac{\pi}{3}\right)\right| = \sqrt{\frac{27}{4}+1} = \boxed{\frac{\sqrt{31}}{2} = v\left(\frac{\pi}{3}\right)}$

203 §13.4 #s 10, 14, 15, 19, 26, 46

(10) Find velocity, acceleration and speed

$\bar{r} = \langle 2\cos t, 3t, 2\sin t \rangle$

$\boxed{\bar{v} = \bar{r}' = \langle -2\sin t, 3, 2\cos t \rangle}$ Constant speed.

$v = |\bar{v}| = \sqrt{4\sin^2 t + 9 + 4\cos^2 t}$ $\boxed{\sqrt{13} = v}$

$\bar{a} = \bar{r}'' = \boxed{\langle -2\cos t, 0, -2\sin t \rangle = \bar{a}}$

(14) Same as #10, $\bar{r} = \langle t^2, \sin t - t\cos t, \cos t + t\sin t \rangle$

$\bar{r}' = \bar{v} = \langle 2t, \cos t - \cos t + t\sin t, -\sin t + \sin t + t\cos t \rangle$

$= \boxed{\langle 2t, t\sin t, t\cos t \rangle = \bar{v}}$

$v = \sqrt{4t^2 + t^2\sin^2 t + t^2\cos^2 t}$

$= \sqrt{4t^2 + t^2} = \boxed{\sqrt{5}\,|t|} = v \quad = \sqrt{5}\,t$ if $t \geq 0$

$= -\sqrt{5}\,t$ " $t < 0$

$\bar{a} = \boxed{\bar{v}'' = \langle 2, \sin t + t\cos t, \cos t - t\sin t \rangle}$

203 § 13.4 #s 15, 19, 26, 46

(15) Find $\bar{v}$, $\bar{r}$, given following info about $\bar{a}$ and position.

$\bar{a} = \langle 1,2,0 \rangle \quad \bar{v}(0) = \langle 0,0,1 \rangle$

$\bar{r}(0) = \langle 1,0,0 \rangle$

$$\bar{v} = \int \bar{a}\,dt + \bar{C} = \int \langle 1,2,0 \rangle\,dt + \bar{C}$$

$$= \langle t, 2t, 0 \rangle + \bar{C}$$

$\bar{v}(0) = \langle 0,0 \rangle + \bar{C} = \langle 0,0,1 \rangle \Longrightarrow \bar{C} = \langle 0,0,1 \rangle$

$\rightarrow \bar{v}(t) = \langle t, 2t, 1 \rangle \Longrightarrow$

$$\bar{r} = \int \langle t, 2t, 1 \rangle\,dt + \bar{C}_2 = \langle \tfrac{1}{2}t^2, t^2, t \rangle + \bar{C}_2$$

$\bar{r}(0) = \langle 0,0,0 \rangle + \bar{C}_2 = \langle 1,0,0 \rangle \longrightarrow \bar{C}_2 = \langle 1,0,0 \rangle$

$\rightarrow \boxed{\bar{r} = \langle \tfrac{1}{2}t^2 + 1, t^2, t \rangle}$

203 § 13.4, 19, 26, 46

(19) $\vec{r}(t) = \langle t^2, 5t, t^2 16t \rangle$ describes position of a particle. Minimize v.

$\bar{v} = \langle 2t, 5, 2t - 16 \rangle \longrightarrow$

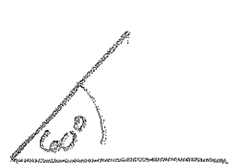
$v = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256}$

$= \sqrt{8t^2 - 64t + 281}$ to be minimized

$\longrightarrow$ Find vertex of $8t^2 - 32t + 281 = y$

$y' = 16t - 64 \overset{\text{SET}}{=} 0 \longrightarrow \boxed{t = 4}$

and $\boxed{v = \sqrt{153} = 3\sqrt{17}}$ there

(26) 

Gun @ 60° from horizontal. Find muzzle speed, if max height of shell is 500 m.

$\vec{r}(t) = \langle v \cos 60^\circ t, (v \sin 60^\circ)t - \frac{1}{2} g t^2 \rangle$

$= \langle \frac{1}{2} v t, \frac{\sqrt{3}}{2} v t - \frac{1}{2} g t^2 \rangle =$

$\vec{r}\,'(t) = \langle \frac{1}{2} v, \frac{\sqrt{3}}{2} v - g t \rangle$ Find t for when vertical component of velocity is 0.

203 §13.4 #s 26, 46

26 Cont'd

SET $\frac{\sqrt{3}}{2}v - gt = 0 \longrightarrow$

$\frac{\sqrt{3}}{2}v = gt \longrightarrow$

$t = \frac{\sqrt{3}\,v}{2g}$. Now, use this to find

when $\bar{r}_y(t) = 500$?

$\bar{r}_y(t) = \frac{\sqrt{3}}{2}vt - \frac{1}{2}gt^2 \overset{SET}{=} 500$

$t = \frac{\sqrt{3}\,v}{2g} \Longrightarrow$

$\frac{\sqrt{3}}{2}v\,\frac{\sqrt{3}\,v}{2g} - \frac{1}{2}g\left(\frac{\sqrt{3}\,v}{2g}\right)^2$

$= \frac{3v^2}{4g} - \frac{1}{2}g\left(\frac{3v^2}{4g^2}\right)$

$= \frac{3}{4g}v^2 - \frac{3gv^2}{8g^2} = 500 \longrightarrow$

$v^2\left(\frac{3}{4g} - \frac{3}{8g}\right) = \frac{3}{8g}v^2 = 500$

$v^2 = \frac{4000g}{3} \Longrightarrow v = \sqrt{\frac{4000g}{3}} \approx \sqrt{\frac{4000(9.8)}{3}}$

$\approx 114.3095212 \frac{m}{s}$

Perfect! If it was 60°!

sigh

203 § 13.4 #s 26, 46

26 Let's get this right! $\theta = 30^\circ$

$$\bar{r}(t) = \langle (v_0 \cos\theta)t, (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \rangle$$

$$= \langle \tfrac{\sqrt{3}}{2} v_0 t, \tfrac{1}{2} v_0 t - \tfrac{1}{2} g t^2 \rangle$$

$$\bar{v} = \bar{r}'(t) = \langle \underbrace{\tfrac{\sqrt{3}}{2} v_0}_{\text{No air friction!}}, \tfrac{1}{2} v_0 - gt \rangle \overset{\text{SET}}{=} 0 \longrightarrow$$

$$\bar{v}_y = \tfrac{1}{2} v_0 - gt = 0 \Longrightarrow gt = \tfrac{1}{2} v_0$$

$$\Longrightarrow t = \frac{1}{2g} v_0 \longrightarrow$$

$$\bar{r}\left(\frac{1}{2g} v_0\right) = \langle *, 500 \rangle, \text{ i.e.,}$$

$$\bar{r}_y\left(\frac{1}{2g} v_0\right) = 500 \longrightarrow$$

$$\frac{1}{2} v_0 \left(\frac{v_0}{2g}\right) - \frac{1}{2} g \left(\frac{v_0}{2g}\right)^2 = 500$$

$$\Longrightarrow \frac{v_0^2}{4g} - \frac{v_0^2}{8g} = 500 \Longrightarrow$$

$$\frac{v_0^2}{8g} = 500 \Longrightarrow v_0^2 = 4000g$$

$$v_0 = \sqrt{4000g} \approx \sqrt{4000(9.8)} \approx 197.9898987$$

$$\approx \boxed{198 \text{ m/s}}$$