§13.3 # 5 1, 4, 7, 10, 13, 15, 17, 21, 28, 47, 50

$ds = |\bar{r}'(t)|\, dt = \sqrt{(x')^2 + (y')^2 + (z')^2}$

$L = \int_\alpha^\beta ds$

(1) Find length of curve

$\bar{r}(t) = \langle t, 3\cos t, 3\sin t \rangle \quad -5 \le t \le 5$

$\Rightarrow \bar{r}'(t) = \langle 1, -3\sin t, 3\cos t \rangle$

$\Rightarrow |\bar{r}'(t)| = \sqrt{1 + 9\sin^2 t + 9\cos^2 t}$

$= \sqrt{1+9} = \sqrt{10}$

$L = \int_{-5}^{5} \sqrt{10}\, dt = \sqrt{10}\, t \Big]_{-5}^{5} = 5\sqrt{10} - (-5\sqrt{10})$

$= \boxed{10\sqrt{10}}$

(4) $\bar{r}(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle \quad 0 \le t \le \frac{\pi}{4}$

$\bar{r}'(t) = \langle -\sin t, \cos t, -\tan t \rangle$

$|\bar{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t}$

$= \sqrt{1 + \tan^2 t} = \sqrt{\sec^2 t} = |\sec t|$

$= \sec t \Rightarrow L = \int_0^{\frac{\pi}{4}} \sec t\, dt =$

§13.3 #s 4, 7, 10, 13, 15, 17, 21, 28, 47, 50

(4) cont'd

$$\int_0^{\frac{\pi}{4}} \sec t \left(\frac{\sec t + \tan t}{\sec t + \tan t}\right) dt$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{\sec^2 t + \sec t \tan t}{\tan t + \sec t}\right) dt$$

$u = \tan t + \sec t$

$\Rightarrow du = \sec^2 t + \sec t \tan t$

$$= \int_{t=0}^{t=\frac{\pi}{4}} \frac{du}{u} = \ln |u| \Big]_{t=0}^{t=\frac{\pi}{4}}$$

$$= \ln |\tan t + \sec t| \Big]_0^{\frac{\pi}{4}} =$$

$$= \ln |1 + \sqrt{2}| - \ln |0 + 1|$$

$$= \ln |1 + \sqrt{2}| = \ln(\sqrt{2} + 1)$$

(7) Find L to 4 decimal places

$\vec{r}(t) = \langle t^2, t^3, t^4 \rangle, \ 0 \le t \le 2$

$\vec{r}\,' = \langle 2t, 3t^2, 4t^3 \rangle$

$|\vec{r}\,'| = \sqrt{4t^2 + 9t^4 + 16t^6}$

$$L = \int_0^2 \sqrt{16t^6 + 9t^4 + 4t^2}\, dt$$

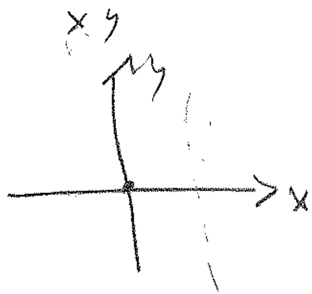
$\approx \boxed{18.6833}$?

§ 13.3 #5, 10, 13, 15, 17, 21, 28, 47, 50

(10) Graph. Find total length of curve, to 4 places

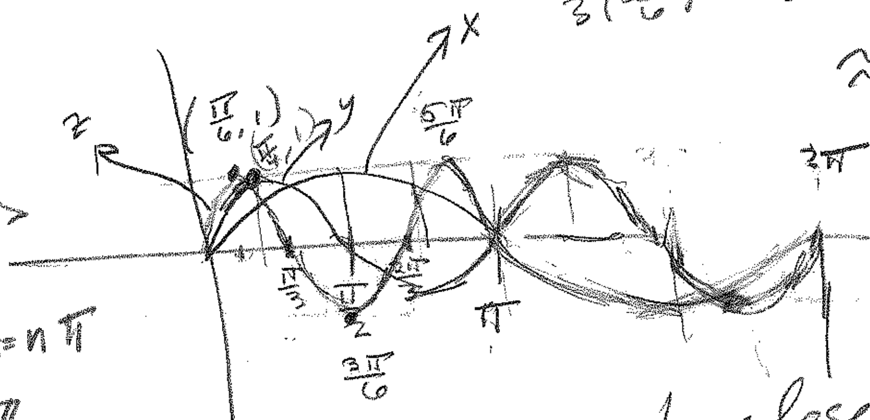
$\bar{r}(t) = \langle \sin t, \sin(2t), \sin(3t) \rangle$

Gets back to starting point @ $t = 2\pi$



$3\left(\frac{5\pi}{6}\right) = \frac{5\pi}{2} = \frac{4\pi}{2} + \frac{\pi}{2}$

$\approx \frac{\pi}{2}$



$\langle x, y, z \rangle = \langle 0, 0, 0 \rangle$

@ $t = 0$, $t = \pi$

~~basically~~ $t = n\pi$

$n \in \mathbb{Z}$

Basically takes $t = 0$ to $t = \pi$ to close (2π) the loops.

t	x	y	z
0	0	0	0
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	0
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	0
$\frac{\pi}{2}$	1	0	-1
$\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	0

t	x	y	z
$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	1
π	0	0	0

$\frac{\cancel{3}(5\pi)}{\cancel{6}_2} = \frac{5\pi}{2}$

§ 13.3 #s 10, 13, 15, 17, 21, 28, 47, 50

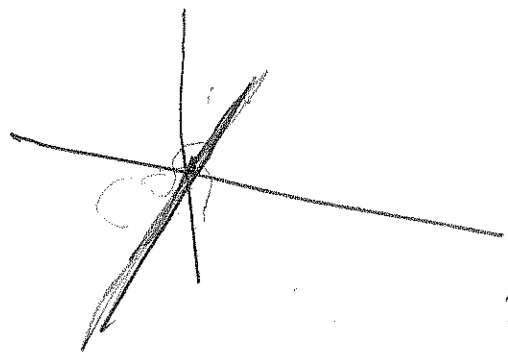
(10) cnt'd

$0 \quad \frac{\pi}{2} \quad \pi$

So x goes from 0 to 1 to 0

z goes from $\underbrace{0 \text{ to } 1}_{t=\frac{\pi}{6}}$ to -1 up to 1 back to 0

$t = \frac{\pi}{2} \qquad t = \frac{5\pi}{6} \qquad t = \pi$

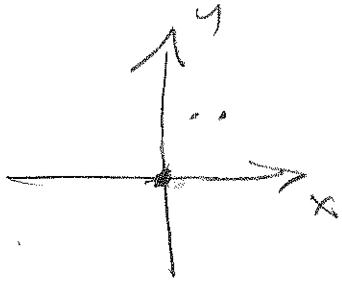


It does some weird bent wire thing, with z oscillating from $z = 1$ to $z = -1$

x

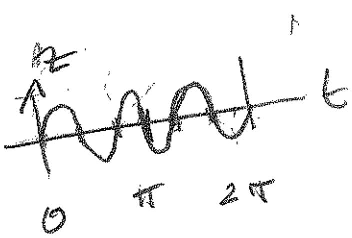
$x = \sin t$

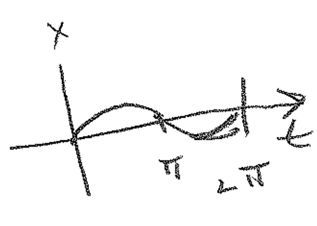
$y = \sin(2t)$



Nope They do get back home by $t = \pi$, but there's more of the curve, from $t = \pi$ to $t = 2\pi$, while the x-values range between 0 & −1 & ... back to zero, while $z = \sin 3t$ does its thing

$3t = 2\pi \rightarrow t = T = \frac{2\pi}{3}$





...5 ~~#~~ 9, 10, 13, 15, 17, 21, 28, 47, 50

(10) cnt'd Bleah.

$$L = \int_0^{2\pi} \sqrt{\cos^2 t + 2^2\cos^2(2t) + 3^2\cos^2(3t)}\, dt$$

$$\approx \boxed{16.0264} \ ?$$

§ 13.3 #s 13, 15, 17, 21, 28, 47, 50

(13) Re-parametrize wrt arc length

$\bar{r}(t) = \langle 2t, 1-3t, 5+4t \rangle$

$\bar{r}(0) = \langle 0, 1, 5 \rangle$

$\bar{r}'(t) = \langle 2, -3, 4 \rangle$

$$s = \int_0^t ds = \int_0^t \sqrt{2^2+3^2+4^2}\,du = \int_0^t \sqrt{29}\,du$$

$s = \sqrt{29}\,t \longrightarrow t = \frac{1}{\sqrt{29}} s \implies$

$$\bar{r}(t(s)) = \left\langle 2\frac{s}{\sqrt{29}},\ 1 - \frac{3s}{\sqrt{29}},\ 5 + \frac{4s}{\sqrt{29}} \right\rangle$$

(15) Start @ (0,0,3), go 5 units along $x = 3\sin t$, $y = 4t$, $z = 3\cos t$ in positive t-direction

where are you now?

$\bar{r}' = \langle 3\cos t, 4, -3\sin t \rangle \implies$

$|\bar{r}'| = \sqrt{9\cos^2 t + 4^2 + 9\sin^2 t} = \sqrt{9+16} = 5$

$s = \int_0^t 5\,du = 5u\Big]_0^t = 5t \implies$

$t = \frac{1}{5}s = \frac{1}{5}(5) = 1 \implies$ you're @ $\boxed{\langle 3\sin(1), 4, 3\cos(1) \rangle}$

given

§13.3 #s 17, 21, 28, 47, 50

(17) Find $\bar{T}$, $\bar{N}$ (a)

$\bar{r} = \langle t, 3\cos t, 3\sin t \rangle$

$\bar{r}' = \langle 1, -3\sin t, 3\cos t \rangle$

$|\bar{r}'| = \sqrt{1^2 + 9\sin^2 t + 9\cos^2 t} = \sqrt{10}$

$\bar{T} = \dfrac{\bar{r}'}{|\bar{r}'|} = \boxed{\dfrac{1}{\sqrt{10}} \langle 1, -3\sin t, 3\cos t \rangle} = \bar{T}'$

$\bar{T}' = \dfrac{1}{\sqrt{10}} \langle 0, -3\cos t, -3\sin t \rangle$

$|\bar{T}'| = \dfrac{1}{\sqrt{10}} \sqrt{9} = \dfrac{3}{\sqrt{10}} \Longrightarrow$

$\bar{N} = \dfrac{\bar{T}'}{|\bar{T}'|} = \dfrac{\sqrt{10}}{3} \left(\dfrac{1}{\sqrt{10}} \right) \langle 0, -3\cos t, -3\sin t \rangle$

$= \langle 0, -\cos t, -\sin t$

w13.3 #s 17, 21, 28, 47, 50

17 b) Use formula $\boxed{9}$ to find curvature.

$$\kappa(t) = \frac{|\bar{T}'(t)|}{|\bar{r}'(t)|} = \frac{\frac{3}{\sqrt{10}}}{\sqrt{10}} = \frac{3}{\sqrt{10}} \cdot \frac{1}{\sqrt{10}} = \boxed{\frac{3}{10}}$$

21) Use $\boxed{T10}$ to find κ.

$\bar{r} = \langle 0, t^3, t^2 \rangle \Rightarrow$

$\bar{r}' = \langle 0, 3t^2, 2t \rangle \Rightarrow$

$\bar{r}'' = \langle 0, 6t, 2 \rangle \Rightarrow$

$|\bar{r}'|^3 = \sqrt{(3t^2)^2 + (2t)^2}^{\,3} = \sqrt{9t^4 + 4t^2}^{\,3}$

$$\begin{array}{rl} \bar{r}' & \langle 0, 3t^2, 2t \rangle \\ \times\, \bar{r}'' & \times\, \langle 0, 6t, 2 \rangle \\ \hline \end{array}$$

$\langle 6t^2 - 12t^2, -0, 0 \rangle = \langle -6t^2, 0, 0 \rangle = \bar{r}' \times \bar{r}''$

$\Rightarrow |\bar{r}' \times \bar{r}''| = \sqrt{36t^4} = 6t^2 \Rightarrow$

$$\frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3} = \boxed{\frac{6t^2}{\sqrt{9t^4+4t^2}^{\,3}} = \kappa(t)}$$

§13.3 #s 28, 47, 50

(28) Use formula $\boxed{11}$ to find κ.

$y = \tan x = f(x)$ $\quad \boxed{11}$ $\kappa(x) = \dfrac{|f''(x)|}{[1+f'(x)^2]^{3/2}}$

$f'(x) = \sec^2 x$

$f''(x) = 2\sec x\,(\sec x \tan x) = 2\sec^2 x \tan x$

$|f''(x)| = |2\sec^2 x \tan x| = 2\sec^2 x\,|\tan x|$

$\Rightarrow$

$1 + f'(x)^2 = 1 + \sec^4 x$

$\Rightarrow \kappa(x) = \dfrac{|f''(x)|}{(1+(f'(x))^2)^{3/2}} = \boxed{\dfrac{2\sec^2 x\,|\tan x|}{(1+\sec^4 x)^{3/2}} = \kappa(x)}$

(47) Find $\bar{T}, \bar{N}, \bar{B}$

$\bar{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$ @ $\langle 1, \frac{2}{3}, 1\rangle$

$\bar{r}'(t) = \langle 2t, 2t^2, 1\rangle$

$|\bar{r}'(t)| = \sqrt{4t^2 + 4t^4 + 1}$

$\bar{T} = (4t^2+4t^4+1)^{-\frac{1}{2}} \langle 2t, 2t^2, 1\rangle$

$\Rightarrow \bar{T}' = -\frac{1}{2}(4t^2+4t^4+1)^{-\frac{3}{2}}(8t+16t^3)\langle 2t, 2t^2, 1\rangle$

$+ (4t^2+4t^4+1)^{-\frac{1}{2}} \langle 2, 4t, 0\rangle$

$= -\frac{1}{2}(4t^2+4t^4+1)^{-\frac{3}{2}}\Big[\langle 16t^2+32t^4, 16t^3+32t^5, 8t+16t^3\rangle$

$-2(4t^2+4t^4+1)\langle 2, 4t, 0\rangle\Big]$

$\rightarrow$ 13.3 # 547, 50

$$= -\tfrac{1}{2}(4t^2+4t^4+1)^{-\frac{3}{2}}\Big[\langle 16t^2+32t^4, 16t^3+32t^5, 8t+16t^3\rangle$$

$$-2\langle 8t^2+8t^4+2, 16t^3+16t^5+4t, 0\rangle$$

$$= -\tfrac{1}{2}(4t^2+4t^4+1)^{-\frac{3}{2}}\Big[\langle 16t^2+32t^4, 16t^3+32t^5, 8t+16t^3\rangle$$

$$-\langle 16t^2+16t^4-4, 32t^3+32t^5+8t, 0\rangle\Big]$$

$$= -\tfrac{1}{2}(4t^2+4t^4+1)^{-\frac{3}{2}}\Big[\langle 16t^4-4, -16t^3-8t, 8t+16t^3\rangle$$

$$\Rightarrow |\overline{T}'| = \frac{1}{2(4t^2+4t^4+1)^{3/2}}\sqrt{(16t^4-4)^2+(16+8t)^2+(8t+16t^3)^2}$$

$$\Rightarrow \overline{N}(1) = \frac{\overline{T}'(1)}{|\overline{T}'(1)|}$$

$$\overline{T}'(1) = -\tfrac{1}{2}(9)^{-\frac{3}{2}}\langle 16-4, -16-8, 8+16\rangle = -\tfrac{1}{2}\left(\tfrac{1}{27}\right)(6)\langle 2,-4,4\rangle$$

$$= -\tfrac{1}{2}(3)^{-3}\langle 12,-24,24\rangle$$

$$= -\tfrac{1}{9}\langle -2,-4,4\rangle = \tfrac{2}{9}\langle 1,2,-2\rangle$$

$$|\overline{T}'(1)| = \tfrac{2}{9}\sqrt{1^2+2^2+2^2} = \tfrac{2}{9}\sqrt{9} = \tfrac{2}{9}\cdot 3 = \tfrac{2}{3}$$

$$\therefore \overline{N}(1) = \frac{\tfrac{2}{9}\langle 1,2,-2\rangle}{\tfrac{2}{3}} = \tfrac{3}{9}\cdot\tfrac{3}{2}\langle 1,2,-2\rangle$$

$$= \tfrac{1}{3}\langle 1,2,-2\rangle$$

§13.3 #47, 50

47 cont'd $\bar{B}(1) = \bar{T} \times \bar{N}$

$\bar{T}(1) = \frac{1}{\sqrt{9}} \langle 2,2,1 \rangle = \frac{1}{3}\langle 2,2,1 \rangle$

$$\begin{array}{rl} \bar{T}(1) & \frac{1}{3}\langle 2,2,1 \rangle \\ \times \bar{N}(1) & \times \frac{1}{3}\langle 1,2,-2 \rangle \\ \hline & \frac{1}{9}\langle -6,-3,2 \rangle \end{array}$$

50 Find eq'ns of normal plane & osculating plane

$\bar{r} = \langle t, t^2, t^3 \rangle$ @ $\langle 1,1,1 \rangle$

$\bar{r}' = \langle 1, 2t, 3t^2 \rangle$ $\qquad \bar{r}'(1) = \langle 1,2,3 \rangle$

$|\bar{r}'| = \sqrt{1+4t^2+9t^4}$ $\qquad \bar{T}(1) = \frac{1}{\sqrt{14}}\langle 1,2,3 \rangle$

$|\bar{r}'(1)| = \sqrt{1+4+9} = \sqrt{14}$

$\bar{T} = \frac{1}{\sqrt{14}}\langle 1, 2t, 3t^2 \rangle$ $\qquad \bar{N}'(1) = \dfrac{\frac{1}{\sqrt{14}}\langle 0,2,6 \rangle}{\frac{1}{\sqrt{14}}\sqrt{40}}$

$\bar{T}' = \frac{1}{\sqrt{14}}\langle 0, 2, 6t \rangle$ $\qquad = \frac{1}{2\sqrt{10}}\langle 0,2,6 \rangle$

$|\bar{T}'| = \frac{1}{\sqrt{14}}\sqrt{4+36t^2}$ $\qquad = \frac{1}{\sqrt{10}}\langle 0,1,3 \rangle = \bar{N}(1)$

$\therefore \bar{B}(1) = \bar{T}(1) \times \bar{N}(1)$

§13.3 #50

50 cont'd

$$\bar{B}(1) = \begin{array}{l} \bar{T}(1) \quad \frac{1}{\sqrt{14}}\langle 1,2,3\rangle \\ \times\ \bar{N}(1) \quad \times\ \frac{1}{\sqrt{10}}\langle 0,1,3\rangle \\ \hline \frac{1}{\sqrt{140}}\langle 3,-(3),1\rangle \end{array}$$

$2\lfloor 140$
$2\lfloor 70$
$5\lfloor 35$
7

$$= \frac{1}{2\sqrt{35}}\langle 3,-3,1\rangle = \bar{B}(1)$$

Normal plane. Contains $\bar{B}, \bar{N}$ & is orthog/perp to $\bar{T}$, so, Let $\bar{x} = \langle x, y, z\rangle$ be p-vec for P on Plane. Then $\bar{x} - \bar{x}_0$ is in P

and $\bar{T}\cdot(\bar{x}-\bar{x}_0) = 0 \longrightarrow$

$$\frac{1}{\sqrt{14}}\langle 1,2,3\rangle \cdot \langle x-1, y-1, z-1\rangle = 0 \longrightarrow$$

$$\frac{1}{\sqrt{14}}(x-1+2(y-1)+3(z-1)) = 0 \longrightarrow$$

$$x-1+2y-2+3z-3 = 0$$

$$\boxed{x+2y+3z=6}$$ Normal Plane.

§13.3 #50

50 Cntd. Now $\overline{B} \perp$ osculating plane,

so $\overline{B} \cdot (\overline{x} - \overline{x}_0) = 0 \Longrightarrow$

$\frac{1}{2\sqrt{35}} \langle 3, -3, 1 \rangle \cdot \langle x-1, y-1, z-1 \rangle = 0$

$3(x-1) - 3(y-1) + z - 1 = 0$

$3x - 3 - 3y + 3 + z - 1 = 0$

$\boxed{3x - 3y + z = 1}$ Osculating Plane.