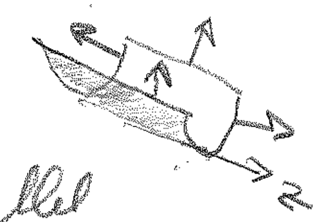
§12.6 #s 1-14, 19

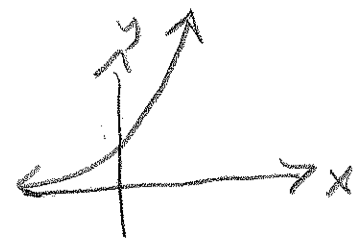
(1) (a) $y=x^2$ is a parabola in $\mathbb{R}^2$

(b) $y=x^2$ " " parabolic cylinder in $\mathbb{R}^3$.
rulings parallel to z-axis

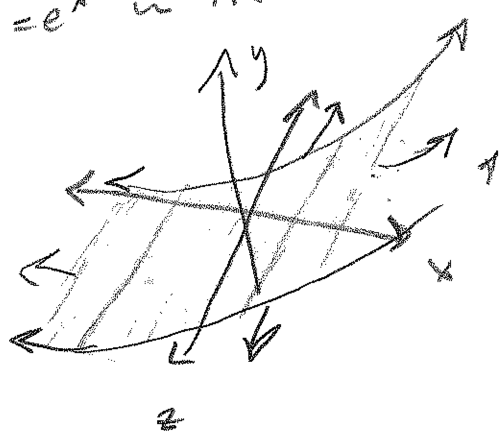


(c) $z=y^2$ same as (b), but rulings parallel to x-axis

(2) (a) $y=e^x$ in $\mathbb{R}^2$:



(b) $y=e^x$ in $\mathbb{R}^3$:



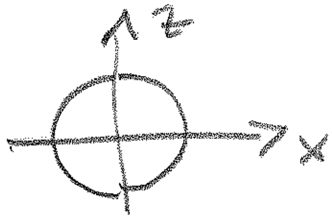
Ruling ∥ to z-axis.
I rotated it to make it easier for me.

(c) $z=e^y$ Take (a) & relabel $y \to z$, $x \to y$
Then take (b) and relabel $z \to x$.

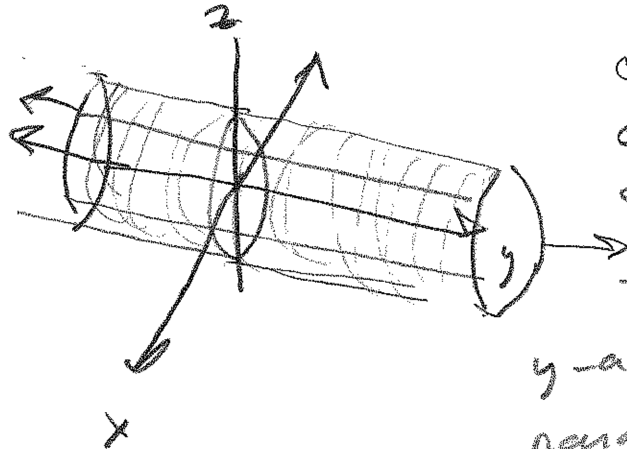
d

→ §12.6 #s 3–14, 19

(3) #s 3–8: Sketch $x^2 + z^2 = 1$ & give description



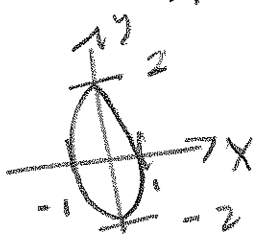
xz-plane

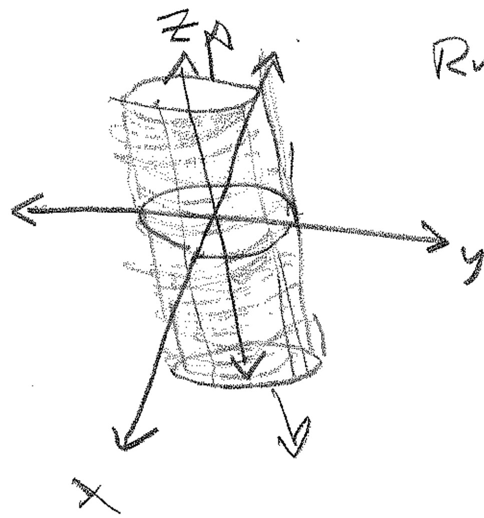


circular cylinder of radius 1, whose axis ~~is~~ is the y-axis & rulings parallel to y-axis

(4) $4x^2 + y^2 = 4$ Similar to previous, only elliptical cross-section, repeating xy-plane projection

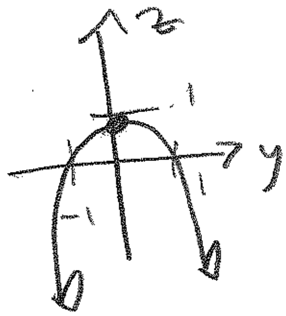
$x^2 + \frac{y^2}{4} = 1$

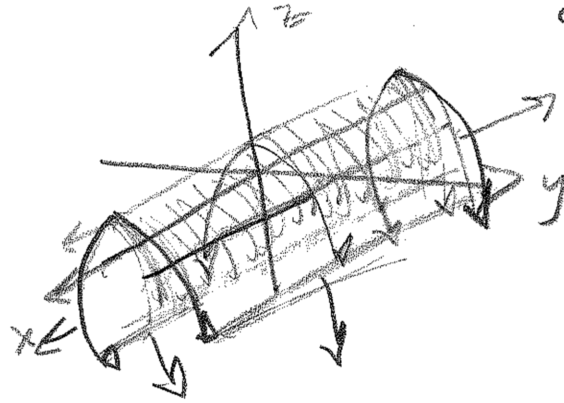




Rulings ∥ to z-axis

(5) $z = 1 - y^2$

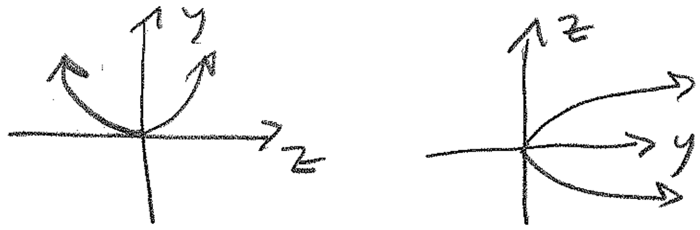


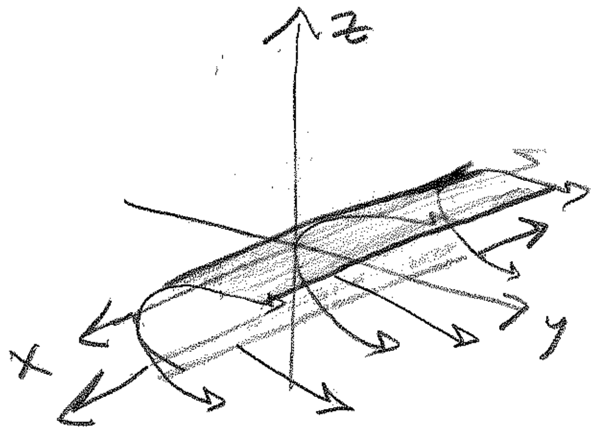


Parabolic cylinder. Rulings ∥ to x-axis.

§12.6 #s 6–14, 19

(6) $y = z^2$

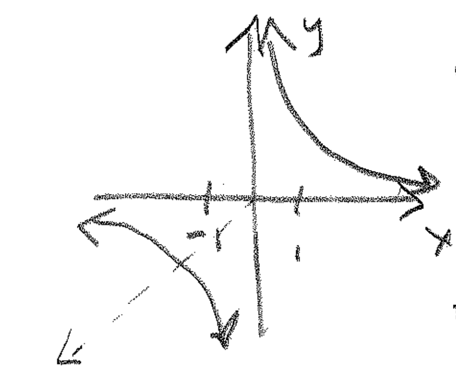




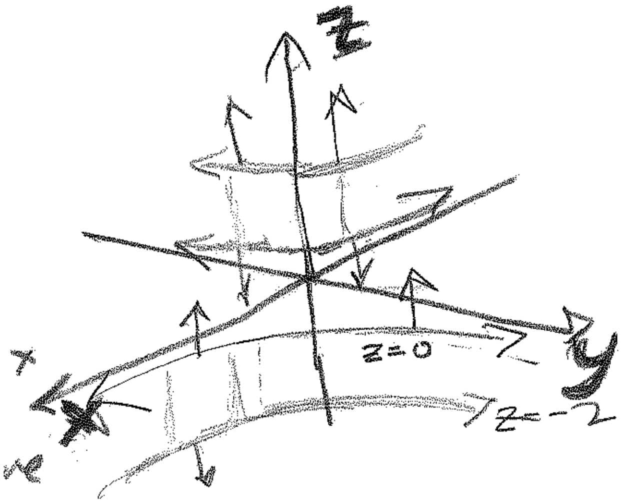
Parabolic cylinder.
Rulings ∥ to x-axis

(7) $xy = 1 \Rightarrow y = \frac{1}{x}$

xy-plane



Conventional
Looking at
xy-plane
from $(0,0,\infty)$

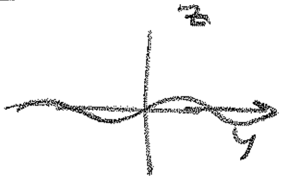


Our eye is on the positive z axis.

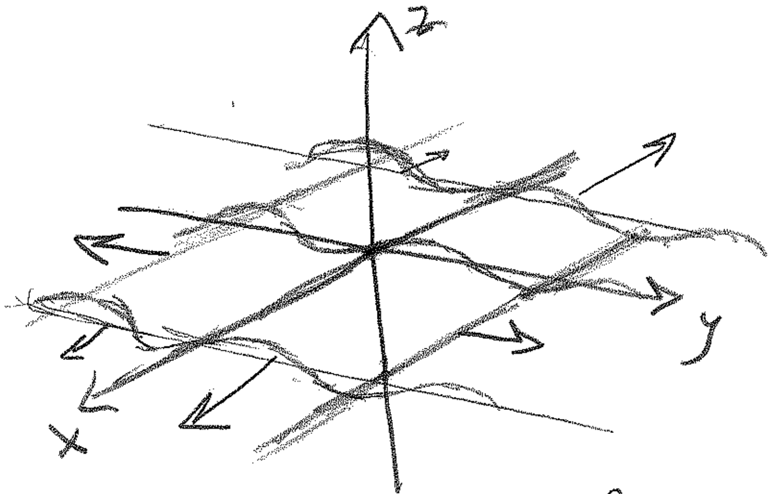
Now the x-axis is going to come towards us, and down, and the z-axis is going to pop into visibility, and rotate

→ §12.6 #s 8–14, 19

(8) $z = \sin y$



sine curve
in yz-plane.



Infinite corrugated siding.

(9) Find and identify the traces of the quadric surface
(a) $x^2 + y^2 - z^2 = 1$, and explain why the graph looks like the hyperboloid of one sheet in TABLE 1. → Keep it around you for reference.

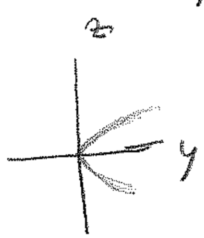
Now we need to be radiologists and piece together cross-sections of 3 types:

$x =$ constant, look @ yz-relation
$y = k$, look at xz-relation
$z = k$, look @ xy-relation

} Basic algorithm for the breakdown

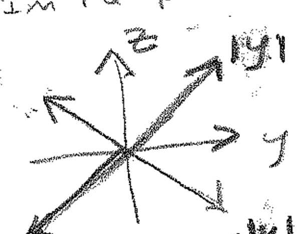
$x = k$ $y^2 - z^2 = 1 - k^2$

Notice when $k > 1$, that right side becomes negative



Bleah

In the plane $x = 1$, this happens:



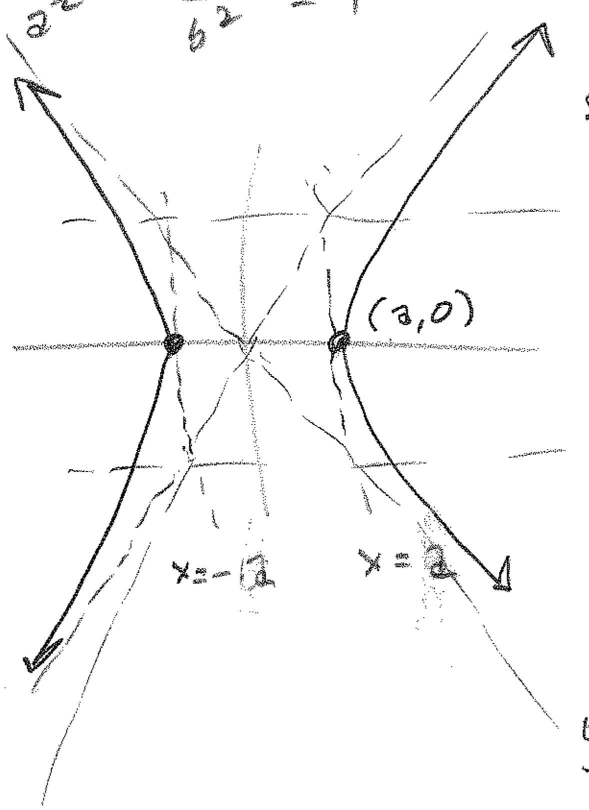
$y^2 = z^2$
$z^2 = y^2$
$z = \pm|y|$

True stuff, but not quite to the point.

$0 < |k| < 1$:
$y^2 - z^2 = 1 - k^2 > 0$
$-z^2 = y^2 + 1 - k^2$
$z = k^2 - 1 + y^2$

§12.6 #s 9-14, 19

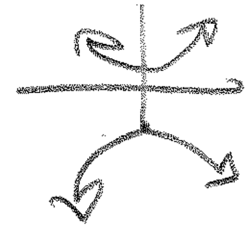
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



Diagonals of the box are slant asymptotes for the hyperbola.

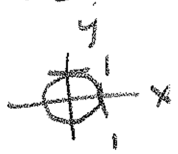
Domain always tells you whether

OR



Now. $x^2+y^2-z^2=1$

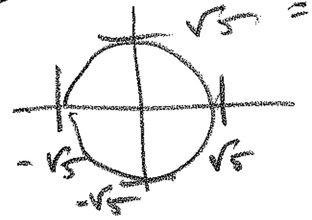
$z=0$



$x^2+y^2=1$

$z=\pm 2$:

$\sqrt{5} = \sqrt{k^2+1}$



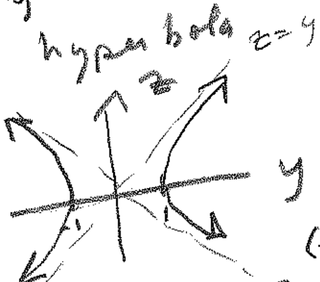
$x^2+y^2=5$

$z=k$: circle radius $= \sqrt{k^2+1}$

$x=0$

$y^2-z^2=1$

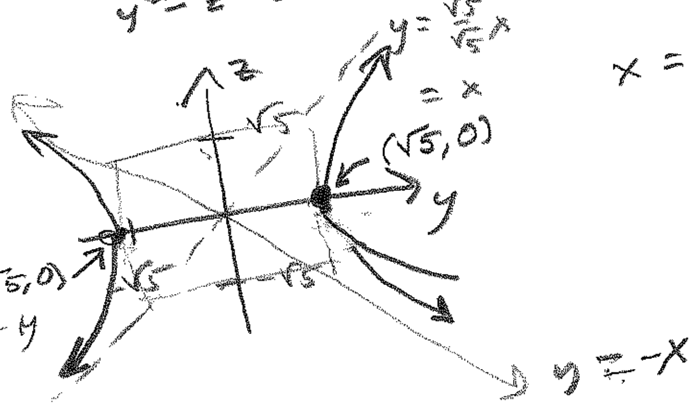
hyperbola



$x=\pm 2$

$y^2-z^2=5 \longrightarrow \frac{y^2}{5}-\frac{z^2}{5}=1$

$x = \pm 5$

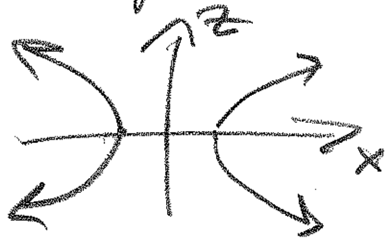


-3 §12.6 #s 9–14, 19

(9) cntd

$y=0$ $x^2-z^2=1$

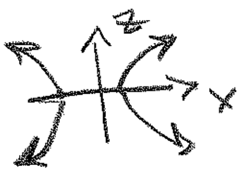
Again a hyperbola, whose "asymptote box" is a square.



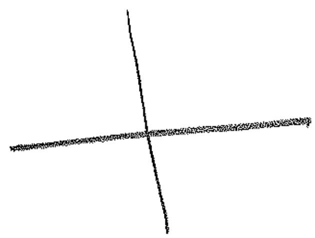
$y=\pm K$

$x^2-z^2=1-K^2$

$0<|K|<1$



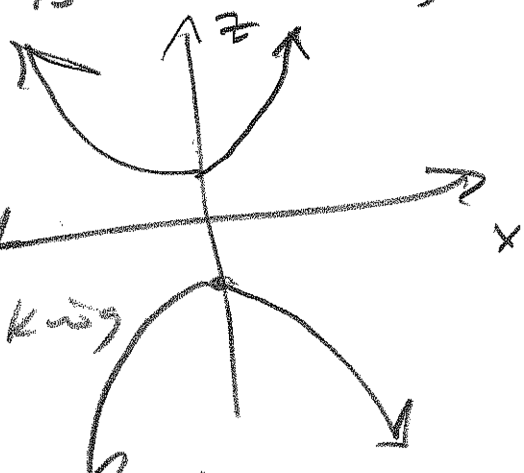
$1<|K|<\infty$



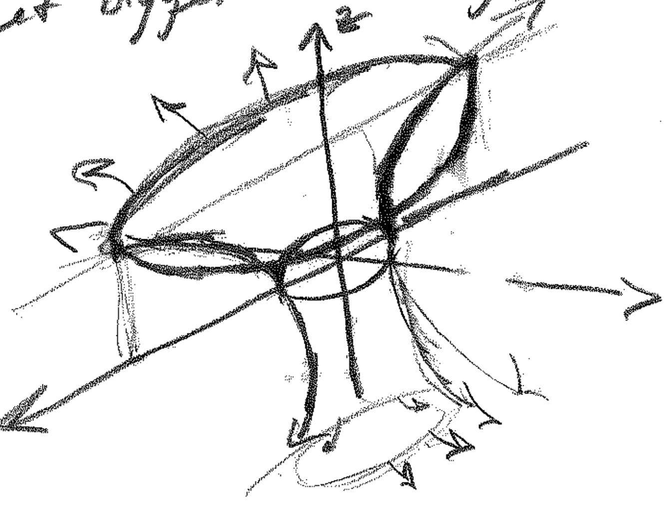
$x^2-z^2=1-K^2<0$

So, $z^2-x^2=K^2-1$

is more plainly



Put these slices all together, you're a better artist than I am. Maybe just stacking the rings from the $z=\pm K$ horizontal slices. Just rings that get bigger and bigger



Its "waist" is at its narrowest cross-section at $z=0$.

→ §12.6 #s 9–14, 19

9b Change part (a) to

$x^2 - y^2 + z^2 = 1$ and it's essentially the same object, only its axis has flipped to lie along the y axis, with cross sections parallel to the y-axis being circles that are getting bigger & bigger as $|y| \to$ Big.

9c Changing part (a) to

$x^2 + y^2 + 2y - z^2 = 0 \longrightarrow$

$x^2 + y^2 + 2y + 1 - z^2 = 1$

$x^2 + (y+1)^2 - z^2 = 1$

This simply moves the $z = \pm K$ cross-sections from circles centered at $(0,0,K)$ to circles centered at $(0,-1,K)$, parallel to the xy-plane!

For the picture, take 9a, & relabel $y=0$ as $y=-1$, so the xz-plane becomes the $y=-1$ plane

√3 §12.6 #s 10–14, 19

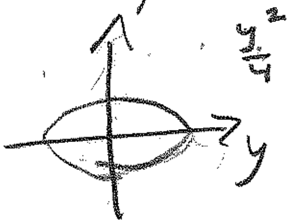
(11) Use traces to sketch and identify the surface.

TABLE 1 says this is an elliptical paraboloid, with elliptical traces in the plane $x = k$, for $k \geq 0$

No trace for $k < 0$!

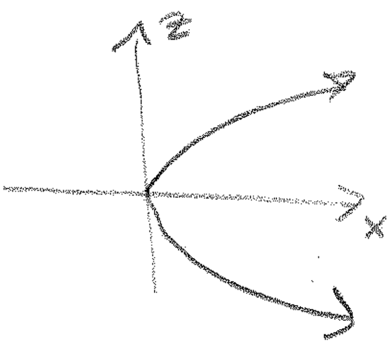
$x = y^2 + 4z^2$

$\frac{y^2}{4} + z^2 = \frac{k}{4}$



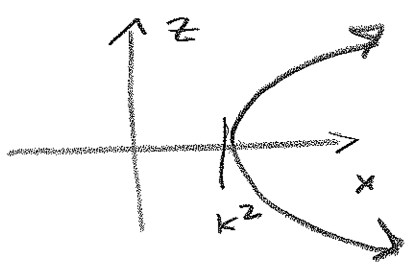
$y = 0$

$x = 4z^2$



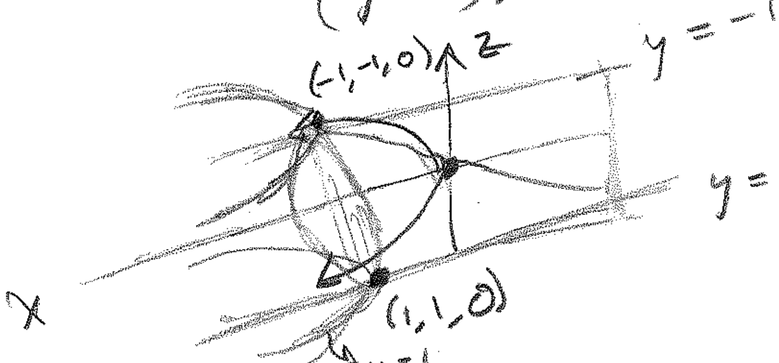
$y = \pm k$

$x = 4z^2 + k^2$ ← y^2



NOTE the vertices are further and further out in the x-direction growing as the square of y

Traces in planes ∥ to xz-plane are parabolas, mirroring each other ~~[illegible]~~ in space, about the xz-plane ($y = 0$).



So its a paraboloid, opening in positive x-direction

3 §12.6 #s 10-14, 19

(10) Use traces to explain why this hyperboloid of 2 sheets, TABLE 1

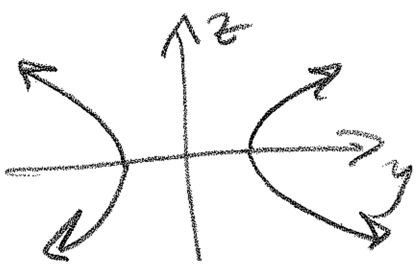
13 (a)

$-x^2-y^2+z^2=1$

$x=\pm k$:

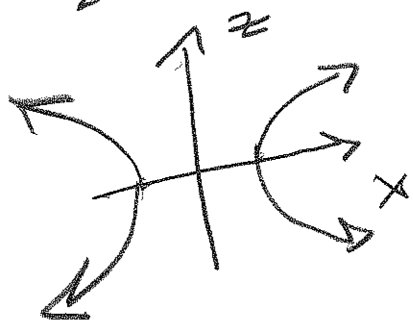
$z^2-y^2=1+x^2$

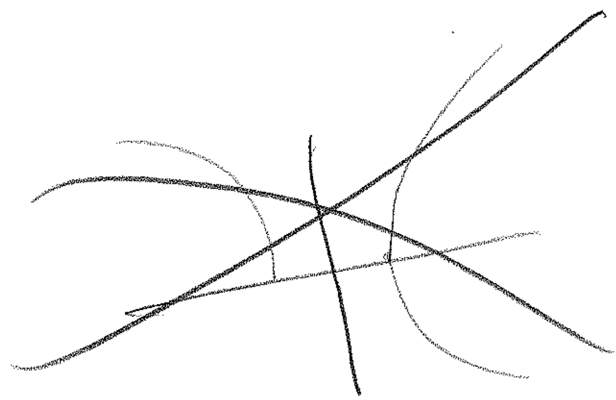
Hyperbolas



$y=\pm k$

$z^2-x^2=y^2+1$ hyperbolas





$z=\pm k$

$-x^2-y^2=1-z^2$

$x^2+y^2=z^2-1$

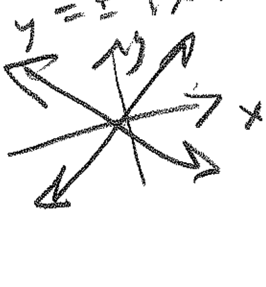
No trace $-1<|z|<1$

$z=1$:

$x^2+y^2=0$

$y^2=x^2$

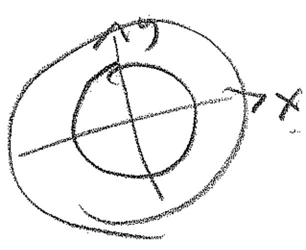
$y=\pm|x|$



$z=\pm k$, $|k|>1$

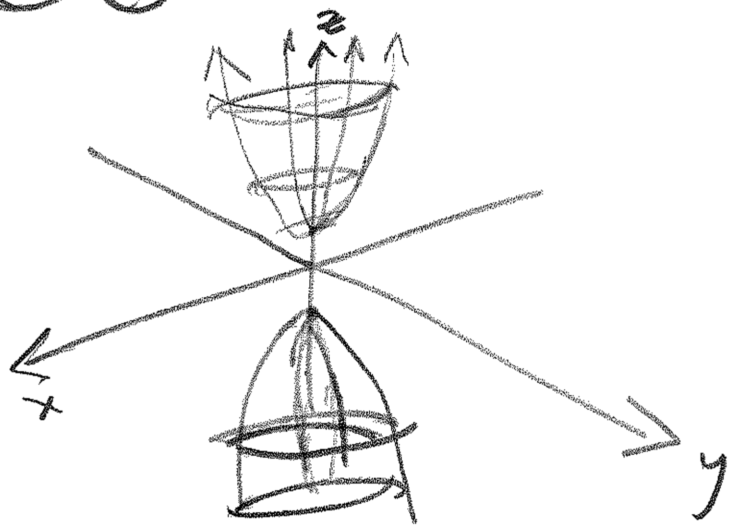
$x^2+y^2=z^2-1$

Circle centered @ $(0,0,k)$ of radius $r=\sqrt{z^2-1}$



.03 §12.6 #s 10–14, 19

10 a central

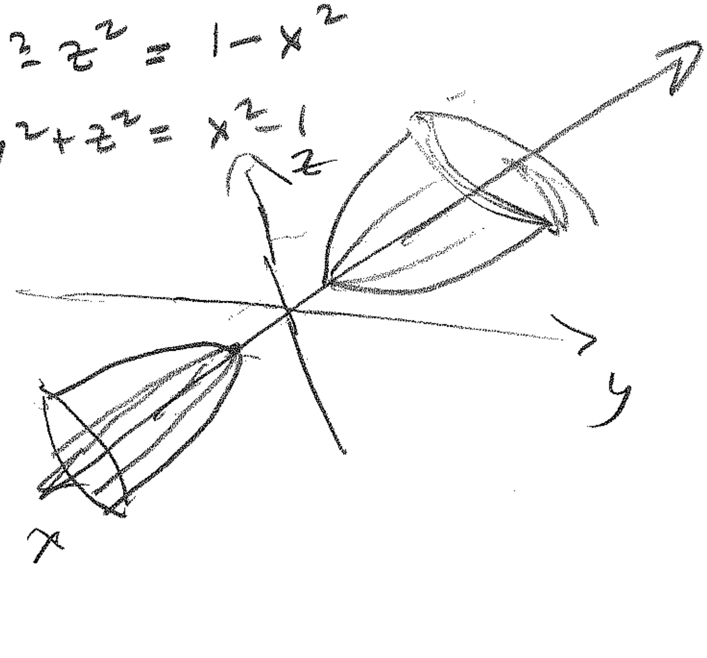


10 b Change (a) to $x^2-y^2-z^2=1$

Same deal, but central axis becomes x-axis. cross-sections parallel to yz-plane are

~~ellipses~~ circles

$$-y^2-z^2=1-x^2$$
$$y^2+z^2=x^2-1$$



-03 § 12.6 #s 12-14, 19

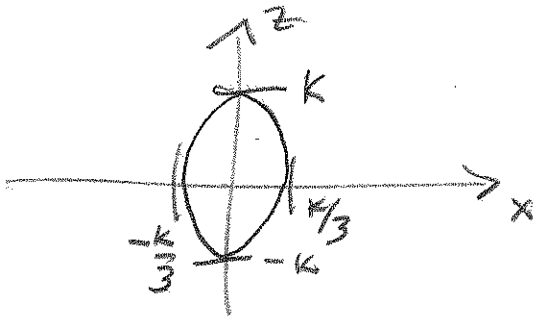
12. Basically #11 for $9x^2 - y^2 + z^2 = 0$

$y = \pm k$ are nice:

$9x^2 + z^2 = k^2$

$\frac{x^2}{\left(\frac{k^2}{9}\right)} + \frac{z^2}{k^2} = 1$

$\frac{x^2}{\left(\frac{k}{3}\right)^2} + \frac{z^2}{k^2} = 1$



Ellipses

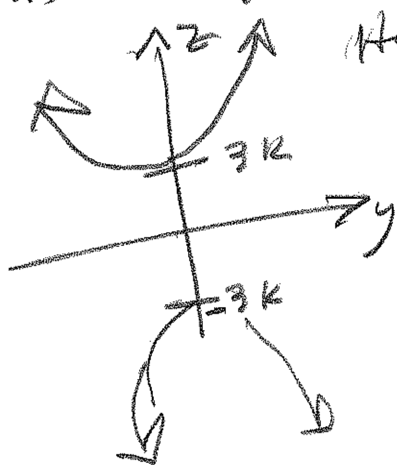
$x = \pm k$

$z^2 - y^2 = -9x^2$

$y^2 - z^2 = 9x^2 = 9k^2 = (3k)^2$

$\frac{y^2}{(3k)^2} - \frac{z^2}{(3k)^2} = 1$

$\underline{k=0 \text{ case}}$: $z = \pm y$ or $\pm|y|$



Hyperbolas in planes $x = \pm k$

The traces ∥ to the yz-plane tells you how these vertices are stacked & offset in space. (so?)

$z = \pm k$

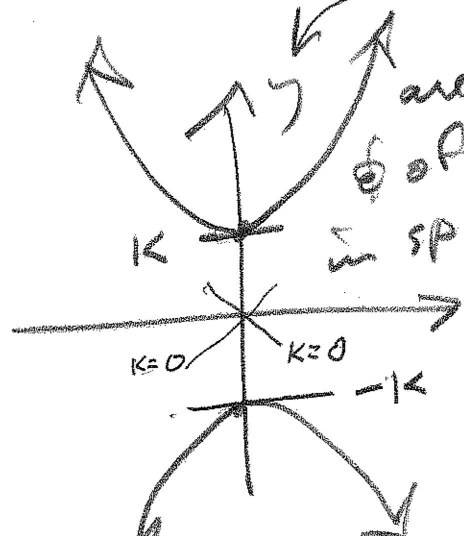
$9x^2 - y^2 = -z^2 = -k^2$

$y^2 - 9x^2 = k^2$

$y^2 - \frac{x^2}{\frac{1}{9}} = k^2$

$\frac{y^2}{k^2} - \frac{x^2}{\left(\frac{k}{3}\right)^2} = 1$

$k = 0$: $y = \pm 3x$ trace in xy-plane.



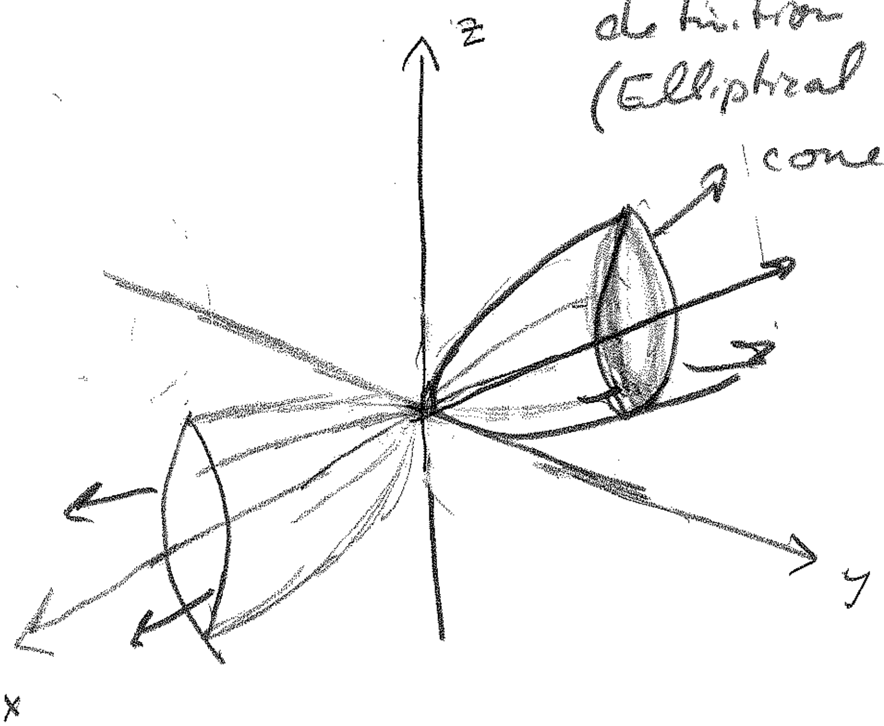
§12.6 #s 12–14, 19

(12) cont'd.

Looks like a paraboloid of 2 sheets

Interesting. Seems like if we look straight down the x-axis, its silhouette is straight-edged.

Oops! That's the definition of a cone! (Elliptical cross-section) cone?



203 §12.6 #s 13, 14, 19

(13) $x^2 = y^2 - 4z^2$

$x^2 - y^2 + 4z^2 = 0$

$x = \pm k$ $\cancel{x = k}$:

$y = \pm k$ $\cancel{y = k}$

$y^2 - 4z^2 = k^2$

$x^2 + 4z^2 = k^2$

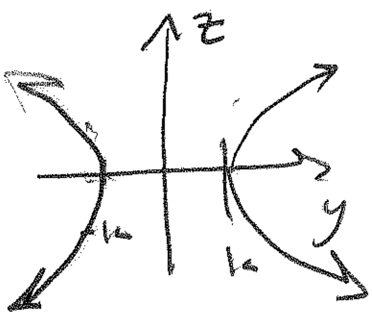
$$\frac{y^2}{k^2} - \frac{z^2}{\left(\frac{k}{2}\right)^2} = 1$$

$k=0$: $z = \pm\frac{1}{2}y$

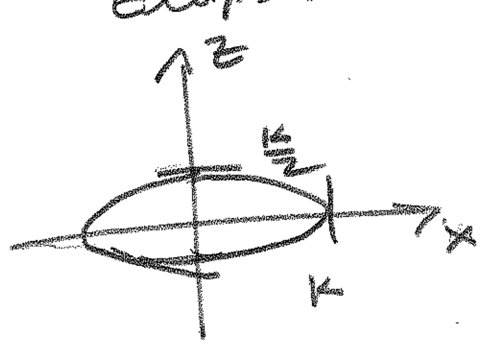
$$\frac{x^2}{k^2} + \frac{z^2}{\left(\frac{k}{2}\right)^2} = 1$$

$k=0$: The point $(0,0,0)$

hyperbolas



Ellipses



Another cone by $k=0$ giving linear traces.

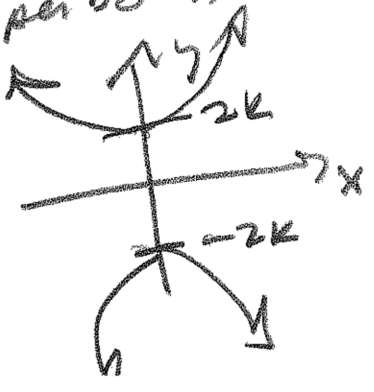
$z = \pm k$

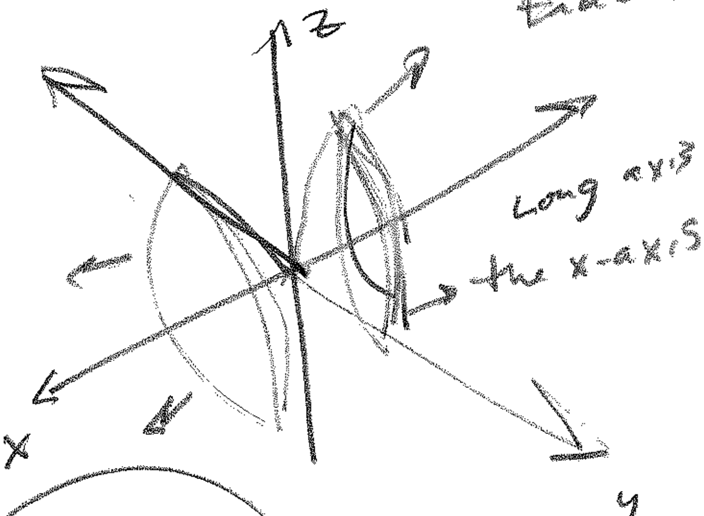
$x^2 - y^2 = -4z^2$

$y^2 - x^2 = 4z^2 = 4k^2$

$$\frac{y^2}{(2k)^2} - \frac{x^2}{(2k)^2} = 1$$

Hyperbolas





$k=0$: $y = \pm x$

$\S$ 12.6 #14, 19

14 $25x^2+4y^2+z^2=100$ Same directions

$x=\pm k$

$4y^2+z^2=100-25x^2$

No expression for

$100-25x^2<0$

$-25x^2<-100$

$x^2>\frac{100}{25}=4$

$|x|>\sqrt{4}=2$

$x>2$ OR $x<-2$

$-2\le x\le 2$:

$4y^2+z^2=100-25k^2$

$$\frac{y^2}{\frac{100-25k^2}{4}}+\frac{z^2}{100-25k^2}=1$$

Ellipses Major axis = 2 times the minor axis

taller in z-direction

$y=\pm k$

$25x^2+z^2=100-4k^2$

No expression for $|k|>5$.

For $|k|\le 5$:

$$\frac{x^2}{\frac{100-4k^2}{25}}+\frac{z^2}{100-4k^2}=1$$

Ellipses. Major axis in z-direction.

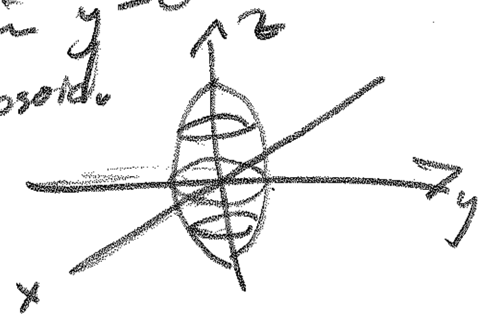
$z=\pm k$:

$25x^2+4y^2=100-z^2$

$$\frac{x^2}{\left(\frac{100-z^2}{25}\right)}+\frac{y^2}{\left(\frac{100-z^2}{4}\right)}=1$$

Ellipses long axis in y-direction

Ellipsoid.



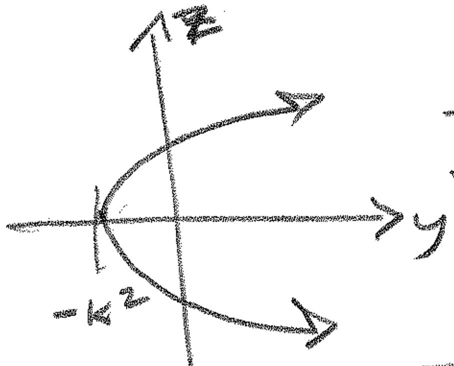
-3 8 12.6 #19

Assume $k > 0$

(19) $y = z^2 - x^2$

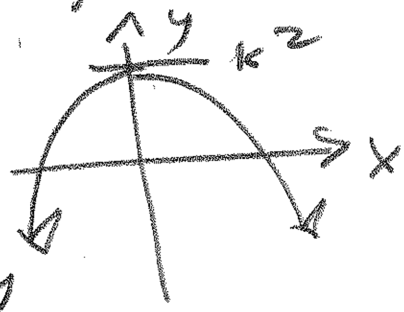
$x = \pm k$

$y = z^2 - k^2$



$z = \pm k$

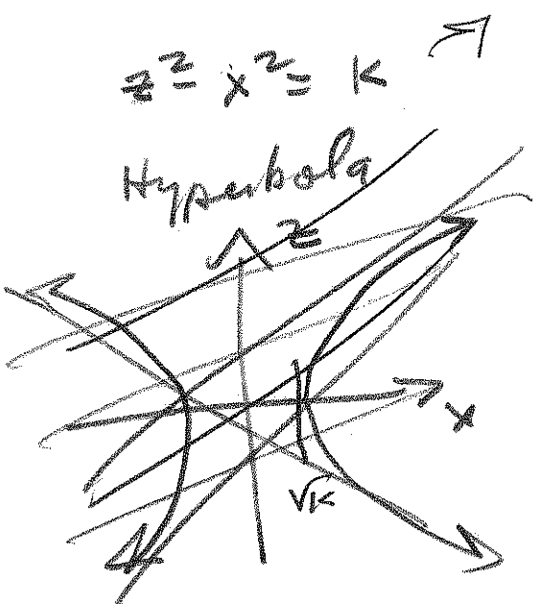
$y = -x^2 + k^2$



$y = k$:

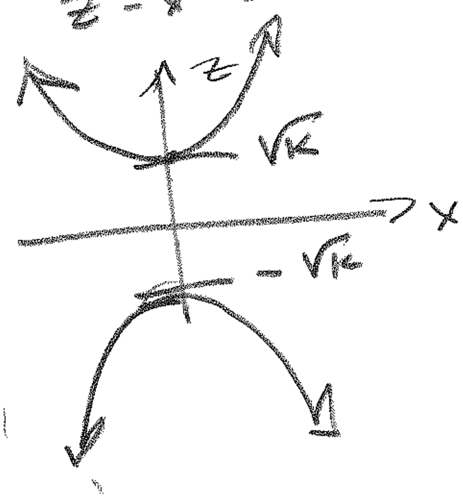
$z^2 - x^2 = k$

Hyperbola



Hyperbolas

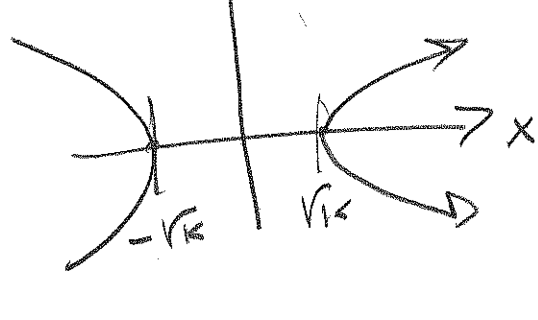
$z^2 - x^2 = k$



$y = -k$

$z^2 - x^2 = -k$

$x^2 - z^2 = k$



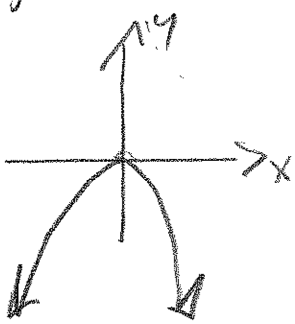
5 §12.6 #19

(19) cntd

xy-plane trace

$z=0$

$y=-x^2$

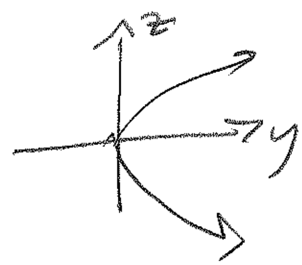


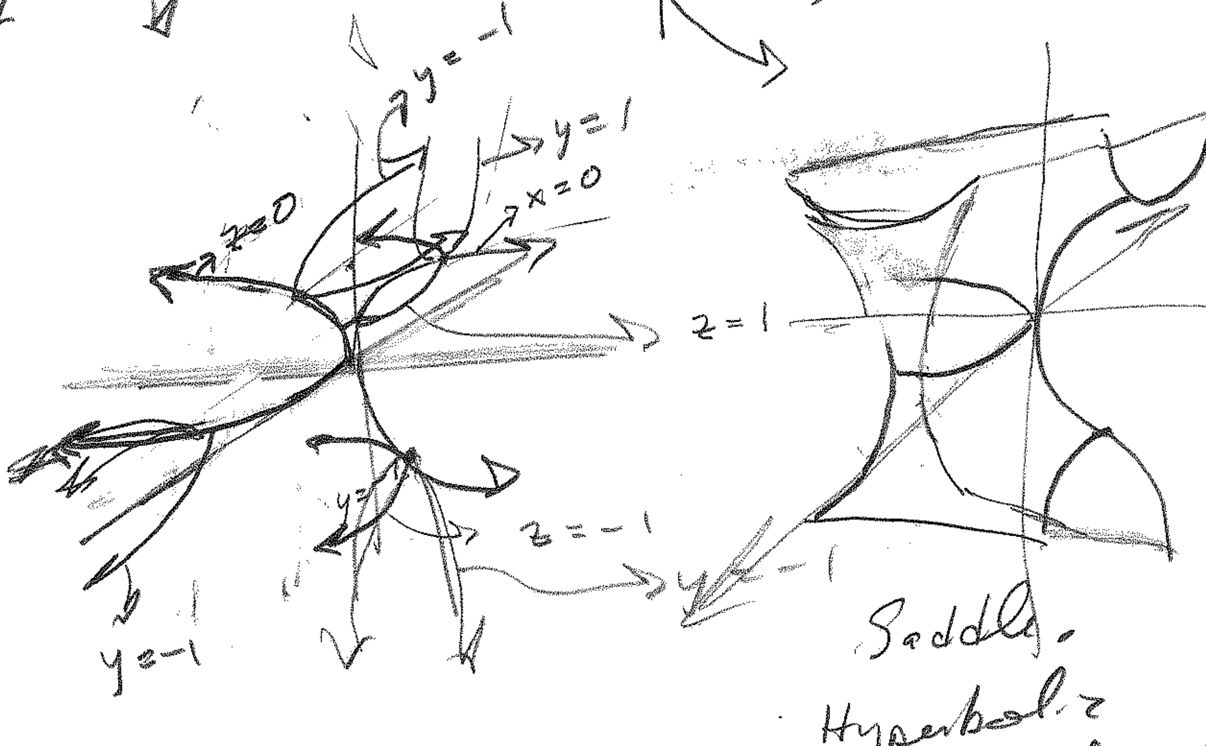
xz-plane $y=0$

$z=\pm x$

yz-plane

$y=z^2$ $x=0$





Saddle.

Hyperbolic

paraboloid.

Very difficult.