§12.5 #s 1-5, 12, 17, 18, 19, 23, 26, 27, 33, 41, 42, 45, 48, 69, 71, 75

1 (a) $l_1 \parallel l_2,\ l_1 \parallel l_3 \Rightarrow l_2 \parallel l_3$ True

(b) $l_1 \perp l_3,\ l_2 \perp l_3 \Rightarrow l_1 \parallel l_2$

True in 2D. Not true in 3-D.

(c) $P_1 \parallel P_3,\ P_2 \parallel P_3 \Rightarrow P_1 \parallel P_2$ True

(d) $P_1 \perp P_3,\ P_2 \perp P_3 \Rightarrow P_1 \parallel P_2$ True

(e) $l_1 \parallel P_1,\ l_2 \parallel P_1 \Rightarrow l_1 \parallel l_2$ FALSE

(f) $l_1 \perp P_1,\ l_2 \perp P_1 \Rightarrow l_1 \parallel l_2$ True

(g) $P_1 \parallel l_1,\ P_2 \parallel l_1 \Rightarrow P_1 \parallel P_2$ FALSE

(h) $P_1 \perp l_1,\ P_2 \perp l_1 \Rightarrow P_1 \parallel P_2$ True

(i) P_1 either intersects P_2 or $P_1 \parallel P_2$ True

(j) l_1 either intersects l_2 or $l_1 \parallel l_2$ FALSE (skew lines)

(k) A plane and a line intersect or they are parallel. True.

§ 12.5 #s 2-5, 12, 17, 18, 23, 26

#s 2-5 Find vector eq'n, symmetric, and parametric eqn's for the line.

(2) ℓ thru $(6,-5,2)$ & $\parallel$ to $\langle 1, 3, -\frac{2}{5} \rangle$

$$\boxed{\vec{x} = \langle 6,-5,2 \rangle + t\langle 1, 3, -\tfrac{2}{5} \rangle, \quad t \in \mathbb{R}}$$

$$\boxed{\begin{aligned} x &= 6 + t \\ y &= -5 + 3t \\ z &= 2 - \tfrac{2}{5}t \end{aligned}}$$

$t = x - 6$

$t = \dfrac{y+5}{3}$

$t = \dfrac{z-2}{-\frac{2}{5}} = \dfrac{5z-10}{-2} = \dfrac{10-5z}{2}$ optional

$$\boxed{x - 6 = \frac{y+5}{3} = \frac{10-5z}{2}}$$

one way to write it.

§ 12.5 #s 3-5, 12, 17, 18, 19, 23, 26, 27, 33, 41, 42,
45, 48, 69, 71, 75

(3) ℓ thru (2, 2.4, 3.5) and $\ell \parallel \langle 3, 2, -1 \rangle$

$$\vec{x} = \begin{bmatrix} 2 \\ 2.4 \\ 3.5 \end{bmatrix} + t \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

MAT 255

$$\boxed{\vec{x} = \langle 2, 2.4, 3.5 \rangle + t \langle 3, 2, -1 \rangle,}$$

MAT 203

$$\boxed{t \in \mathbb{R}}$$

$$\boxed{\begin{aligned} x &= 2 + 3t \\ y &= 2.4 + 2t \\ z &= 3.5 - t \end{aligned}}$$

$$t = \boxed{\frac{x-2}{3} = \frac{y-2.4}{2} = 3.5 - z}$$

(4) ℓ thru (0, 14, -10) & $\ell \parallel$ to $x = -1 + 2t$, $y = 6 - 3t$, $z = 3 + 9t$

vector

$$\cancel{\boxed{\vec{x} = \langle -1, 6, 3 \rangle + t \langle 2, -3, 9 \rangle \quad t \in \mathbb{R}}}$$

NOPE

Symmetric

$$\cancel{\boxed{t = \frac{x+1}{2} = \frac{y-6}{-3} = \frac{z-3}{9}}}$$

$$\boxed{\vec{x} = \langle 0, 14, -10 \rangle + t \langle 2, -3, 9 \rangle, \quad t \in \mathbb{R}}$$

$$\begin{aligned} x &= 2t \\ y &= 14 - 3t \\ z &= -10 + 9t \end{aligned} \implies t = \boxed{\frac{x}{2} = \frac{y-14}{-3} = \frac{z+10}{9}}$$

- §12.5 #s 5, 12, 17, 18, 19, 23, 26, 27, 33, 41, 42, 45, 48, 69, 71, 75

(5) l thru $(1,0,6)$ and $\perp$ to P: $x+3y+z=5$

$\bar{v} = \langle 1, 3, 1 \rangle$ is $\parallel$ to l (direction vector.

So, $\boxed{\bar{x} = \langle 1,0,6\rangle + t\langle 1,3,1\rangle, \ t \in \mathbb{R}}$

$$\boxed{\begin{aligned} x &= 1+t \\ y &= 3t \\ z &= 6+t \end{aligned}} \quad \text{OR} \quad \boxed{t = \frac{x-1}{1} = \frac{y}{3} = \frac{z-6}{1}}$$

(12) The line of intersection between

Give parametric & symmetric eq'ns

$x+2y+3z=1$

& $x-y+z=1$

$\Rightarrow x = y-z+1$

$\Rightarrow x+2y+3z$

$= (y-z+1)+2y+3z = 1$

$\boxed{3y+2z=0}$ Gen'l

Now, we need a direction vector

Find another pt on the line.

$z = 6$

$x = -5(6) + 1 = -29$

$y = -\frac{2}{3}(6) = -4$

so $\bar{r}_1 = \begin{bmatrix} -29 \\ -4 \\ 6 \end{bmatrix}$

and $\bar{r}_1 - \bar{r}_0 = \begin{bmatrix} -29 - (-14) \\ -4 - (-2) \\ 6 - 3 \end{bmatrix} = \begin{bmatrix} -15 \\ -2 \\ 3 \end{bmatrix}$

$= \bar{v}$ is in the right direction.

Vector Eq'n:

$\bar{r} = \bar{r}_0 + t\bar{v} = \begin{bmatrix} -14 \\ -2 \\ 3 \end{bmatrix} + t \begin{bmatrix} -15 \\ -2 \\ 3 \end{bmatrix}$

<u>Parametric Eq'ns:</u>

$x = -14 - 15t$

$y = -2 - 2t$

$z = 3 + 3t$

<u>Symmetric:</u>

$\frac{x+14}{-15} = \frac{y+2}{-2} = \frac{z-3}{3}$

METHOD 2

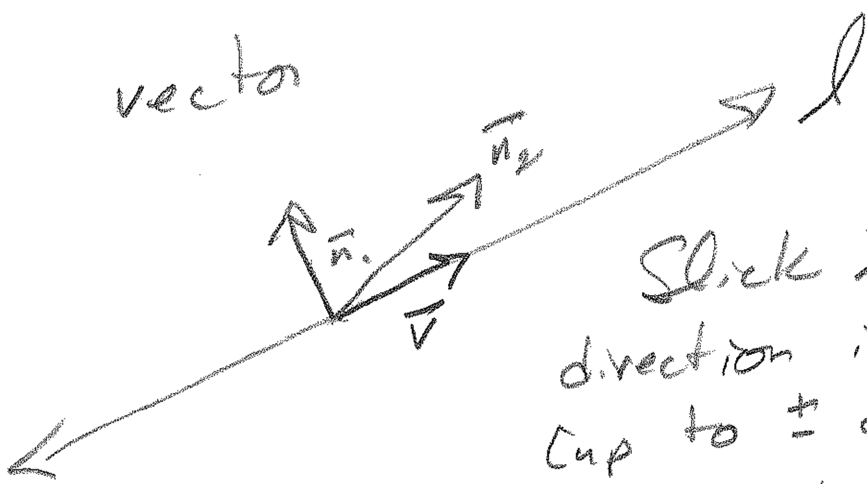
$$x + 2y + 3z = 1 \qquad \bar{n}_1 = \langle 1, 2, 3 \rangle$$

$$x - y + z = 1 \qquad \bar{n}_2 = \langle 1, -1, 1 \rangle$$

$\bar{n}_1$ & $\bar{n}_2$ are perpendicular to their respective planes, so any line of intersection is perpendicular to both, as is

$$\bar{n}_1 \times \bar{n}_2 = \langle 1, 2, 3 \rangle \times \langle 1, -1, 1 \rangle$$

$$= \langle 5, 2, -3 \rangle = \bar{v} = \text{direction vector}$$



Slick! This direction is unique (up to ± direction)

$z = 0$; $\langle 1, 0, 0 \rangle$ works for both,

So

§12.5 #12

Line of intersection for the planes

$x + 2y + 3z = 1$ $\boxed{\text{METHOD 1}}$

$x - y + z = 1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & -1 & 1 & 1 \end{array}\right] \xrightarrow{-R1+R2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -2 & 0 \end{array}\right]$$

$$3R1+2R2 \left[\begin{array}{ccc|c} 1 & 0 & 5 & 1 \\ 0 & -3 & -2 & 0 \end{array}\right]$$

$x + 5z = 1$ $-3y - 2z = 0$

$x = -5z + 1$ $-3y = 2z$

$y = -\frac{2}{3}z$

Let $z = 3$. Then

$x = -5(3) + 1 = -14$ and

$y = -\frac{2}{3}(3) = -2$, so

$\bar{r}_0 = \begin{bmatrix} -14 \\ -2 \\ 3 \end{bmatrix}$ is p-vect. for pt. on line

$\S$ 12.5 #s ~~..~~, 17, 18, 19, 23, 26, 27, 33, 41, 42, 45, 48, 69, 71, 75

(18) Find parametric eqns for the line thru $(2,4,6)$ that's $\perp$ to $x-y+3z=7$

$\bar{n} = \langle 1,-1,3 \rangle$ is $\perp$ l.

is direction vector for l.

DON'T NEED:

$\bar{x} \longleftarrow \langle 2,4,6\rangle = \bar{u}$

$\bar{x}-\bar{u}$ lies in the plane

So

$\bar{x} = \bar{u} + t\bar{n} \implies$ $\boxed{\begin{aligned} x &= 2+t \\ y &= 4-t \\ z &= 6+3t \end{aligned}}$

(17) Find vector eq'n for line segment from $(2,-1,4)$ to $(4,6,1)$

$\bar{u} = \langle 2,-1,4\rangle$, $\bar{v} = \langle 4,6,1\rangle \implies$

$\bar{x} = (1-t)\bar{u} + t\bar{v}$, $0 \le t \le 1$

$= \langle 2(1-t), -1(1-t), 4(1-t)\rangle + \langle 4t, 6t, 1t\rangle$

$= \langle 2-2t+4t, -1+t+6t, 4-4t+t\rangle$

$= \langle 2+2t, -1+7t, 4-3t\rangle$

$\boxed{= \langle 2,-1,4\rangle + t\langle 2,7,-3\rangle \quad 0 \le t \le 1}$

→ §12.5 #s 18, 19, 23, 24, 27, 33, 41, 42, 45, 48, 69, 71, 75

(18) Parametric eq'ns for segment from

$\langle 10, 3, 1\rangle$ to $\langle 5, 6, -3\rangle$

$\langle 10(1-t), 3(1-t), 1(1-t)\rangle + \langle 5t, 6t, -3t\rangle$

$= \langle 10-10t+5t, 3-3t+6t, 1-t-3t\rangle$

$= \langle 10, 3, 1\rangle + \langle -5t, 3t, -4t\rangle$

$$\boxed{\begin{aligned} x &= 10-5t \\ y &= 3+3t \\ z &= 1-4t \end{aligned}}$$

(19) Are L_1, L_2 parallel, skew, or intersecting?

L_1: $\langle 3+3t, 4-t, 1+3t\rangle$ Not $\parallel$.

L_2: $\langle 1+4s, 3-2s, 4+5s\rangle$

x's: