§17.8 #17

$\bar{F}$ along $(0,0,0)$ to $(1,0,0)$ to $(1,2,1)$
$\underbrace{\text{to } (0,2,1) \text{ to } (0,0,0)}$

Line segment is

$\bar{r}(t) = (1-t)\langle 0,2,1\rangle + t\langle 0,0,0\rangle$
$0 \le t \le 1$

$= \langle 0, 2-2t, 1-t\rangle$

$x = 0,\ y = 2-2t,\ z = 1-t$

why can I choose 2 different normals to the plane?

$\text{curl}\,\bar{F} \cdot d\bar{S} = \text{curl}\,\bar{F} \cdot \bar{n}\, dS$

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To the plane that represents the surface between these line segments: $z = \frac{1}{2}y \implies -\frac{1}{2}y + z = 0 \implies y - 2z = 0$ $\langle 0,1,-2\rangle$

$\bar{r}(x,y) = \langle x, y, \frac{1}{2}y\rangle$

$\bar{r}_x \times \bar{r}_y = \langle 0, -\frac{1}{2}, 1\rangle \implies |\bar{r}_x \times \bar{r}_y| = \frac{\sqrt{5}}{2}$

you could use any normal to the plane

$$\boxed{\bar{n} = \frac{\langle 0,1,-2\rangle}{\sqrt{5}}}$$

$$\iint_S \operatorname{curl}\bar{F} \cdot \frac{\langle 0,-1,2\rangle}{\sqrt{5}}\, dS$$

$$\iint_S \operatorname{curl}\bar{F} \cdot \frac{\langle 0,-\frac{1}{2},1\rangle}{\frac{\sqrt{5}}{2}}\, dS$$

$\operatorname{curl}\bar{F} = \langle 8y, 2z, 2y\rangle$

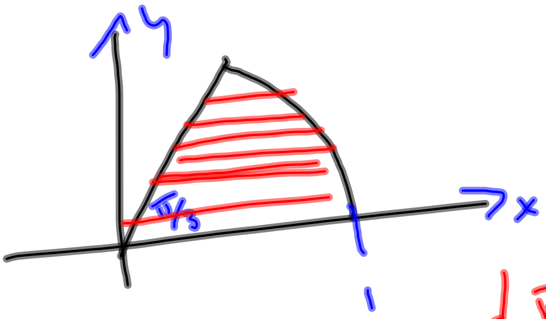
we want to evaluate this as an iterated integral.

dS to dA, where it's now over a rectangle in the xy-plane.

$dS = |\bar{r}_x \times \bar{r}_y|\, dA$

So if you <u>had a brain</u>, you'd use the $\bar{n} = \frac{\bar{r}_x \times \bar{r}_y}{|\bar{r}_x \times \bar{r}_y|}$

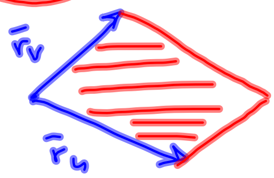
So it would cancel with the Jacobian $|\bar{r}_x \times \bar{r}_y|$ deal that gets us from dS to $dA =$
$= dx\,dy = dy\,dx$.

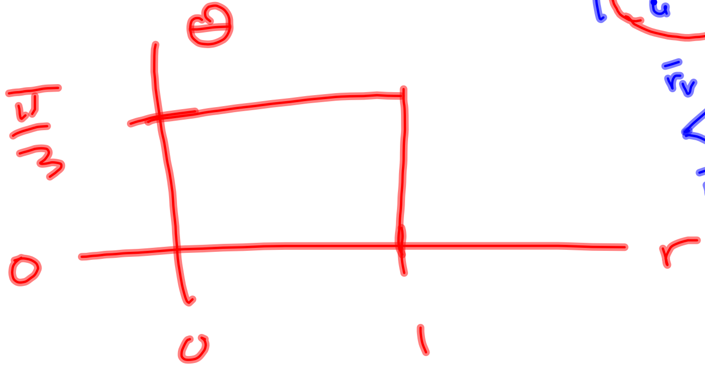


$$\left|\bar{r}_r \times \bar{r}_\theta\right| \rightarrow r\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{3}} \int_0^1$$

$$\left|\bar{r}_u \times \bar{r}_v\right|$$





$\langle 0, -\frac{1}{2}, 1\rangle$ & $\langle 0,-1,2\rangle$ were legit normals, but only the $\langle 0,-\frac{1}{2},1\rangle$ comes from $\bar{F}(x,y) = \langle x, y, \frac{1}{2}y\rangle$ & makes the transition from $\cdot d\bar{S}$ to $\cdot \bar{n}\, dS$ to $\cdot \bar{n}\, dA$, which is $\cdot \bar{n}\, dx\,dy$ for the iterated integral.

17.8.9 thing is the way to reason these out.

Conservative $\bar{F} = \nabla f$ for some $f(x,y,z)$

Building f from $\bar{F}$.

$$\bar{F} = \langle f_x, f_y, f_z\rangle = \nabla f$$

Find f:

$$f = \int f_x\, dx + g(y,z)$$

"Variation of Parameters"

Differentiate wrt y

It'll match up with

$$f_y = \frac{\partial}{\partial y}\left[\int f_x\, dx + g_y(y,z)\right]$$

etc.

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$\int_C \bar{F} \cdot d\bar{r}$ when you parametrize

$C: \bar{r}(t) = \langle x(t), y(t), z(t) \rangle$

along a line segment

or a function $\boxed{\begin{array}{c} z = g(y) \\ \text{or} \\ z = g(x) \end{array}}$

This was this thinking

$\int$ 17.2 #15

Maybe the C_1 part of it.

$\int_C (3x+2y)\,ds$ where c is some $y = f(x)$.

$ds = \sqrt{x_t^2 + y_t^2}\; dt$

$= \sqrt{1 + y_x^2}\; dx$

if x is the parameter.

$(x_x)^2 = 1^2$

Green's

Stokes'

Divergence

what if $y = h(x,z)$?

$\bar{r}(x,z) = \langle x, h(x,z), z \rangle$ &

x & z are dandy ...

17.6 #29

spheres, cylinders, cone

#33

~~17.7.10~~

~~Special where~~

~~$z = g(x,y)$~~

Learn the Vector Calc. approach.

$\boxed{\bar{r} = \langle x, y, g(x,y) \rangle}$ &

x & y are DANDY parameters for

$\boxed{\bar{r}_u \times \bar{r}_v}$ thing.

$d\bar{S}$ to dA

to $dx\,dy$, or

$du\,dv$, or

$dr\,d\theta$

Spherical coords from rectangular:

$\boxed{\rho^2 \sin\phi}\, d\rho\, d\theta\, d\phi$

$|\bar{r}_\phi \times \bar{r}_\theta|$

17.2

$$\int_C P\,dx + Q\,dy + R\,dz$$

$$\int_C P\,dx + Q\,dy = \iint_D (Q_x - P_y)\,dA$$

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