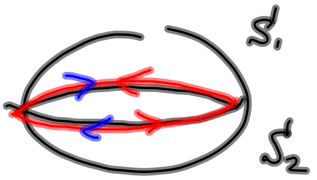
#19
§17.8

$$\iint_S \text{curl}\,\bar{F}\cdot d\bar{S} = 0 \quad \text{if } S \text{ is a sphere.}$$

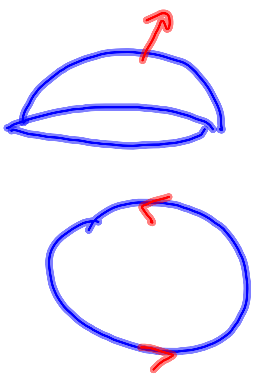
why?



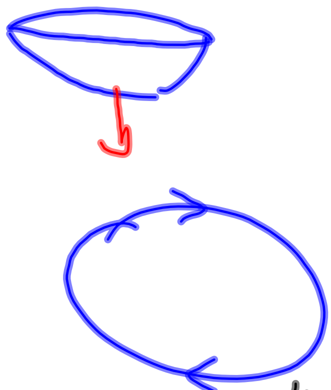
$$\iint_S = \iint_{S_1} + \iint_{S_2} = \int_{C_1} + \int_{C_2} = 0, \text{ because}$$

C_1 & C_2 have opposite orientations.

By Stokes



out is positive
up is positive
outward normal is up



Out is positive
down is positive
outward normal is down.

$$\iint_S \bar{F}\cdot d\bar{S} = \iiint_E \operatorname{div} \bar{F}\, dV$$

Simple Solid Region (Think "convex")

Flux of $\bar{F}$ across $\underbrace{\text{the boundary of } E}_{S}$ is

$F(b)-F(a)$

equal to the triple Integral of the divergence of $\bar{F}$ over E!

$$\int_a^b f'(x)\,dx$$

$$\operatorname{div}\bar{F} = \operatorname{div}\langle P,Q,R\rangle = P_x + Q_y + R_z$$
$$= \nabla\cdot\bar{F}$$

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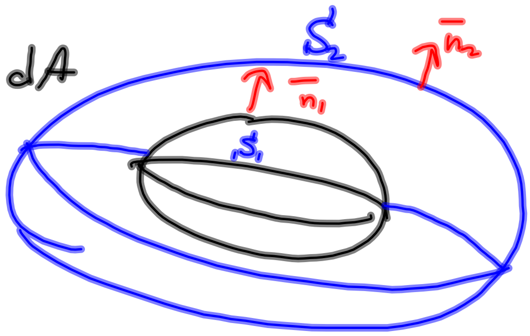
Recall Green's

$$\int_C \bar{F} \cdot \bar{n} \, ds = \iint_D \operatorname{div} \bar{F} \, dA$$

$$= \iint_D (P_x + Q_y) \, dA$$



Eg 7:

$$\boxed{\iiint_E \operatorname{div} \bar{F} \, dV} = \iint_{S_1} \bar{F} \cdot (-\bar{n}_1) \, dS + \iint_{S_2} \bar{F} \cdot \bar{n}_2 \, dS$$

E is the region between these closed surfaces.

On S_1, the outward normal to E is actually in

On S_2, the outward normal to the closed surface is out

$$\iiint_E \operatorname{div} \bar{F}\, dV = \iint_S \bar{F} \cdot d\bar{S}$$

$S = S_1 \cup S_2$, S_1 & S_2 share no points in common, we have

$$\iint_S \bar{F} \cdot d\bar{S} = \iint_{S_1} \bar{F} \cdot d\bar{S} + \iint_{S_2} \bar{F} \cdot d\bar{S}$$

$$\Longrightarrow \iiint_E \operatorname{div} \bar{F} - \iint_{S_1} \bar{F} \cdot d\bar{S} = \iint_{S_2} \bar{F} \cdot d\bar{S}$$

$$\iiint_E \operatorname{div} \bar{F} + \iint_{S_1} \bar{F} \cdot \bar{n}_1\, dS = \boxed{\iint_{S_2} \bar{F} \cdot dS}$$

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$$\bar{E} = \frac{\varepsilon Q}{|\bar{x}|^3}\,\bar{x}$$ on the surface of a sphere of radius a:

$$= \frac{\varepsilon Q}{|\bar{x}|^3}\,\bar{x} = \frac{\varepsilon Q}{(x^2+y^2+z^2)^{3/2}}\langle x, y, z\rangle$$

$$\operatorname{div}\bar{E} = \frac{\partial}{\partial x}\left(\frac{x}{(x^2+y^2+z^2)^{3/2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{(x^2+y^2+z^2)^{3/2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= \underline{(x^2+y^2+z^2)^{-3/2}} + x\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x)$$

$$+ \underline{(x^2+y^2+z^2)^{-3/2}} + y\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}y)$$

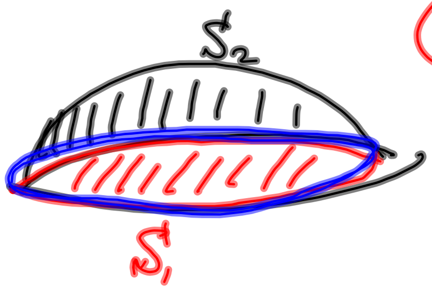
$$+ (x^2+y^2+z^2)^{-3/2} + z\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}z)$$

$$= \frac{x^2+y^2+z^2-3x^2+x^2+y^2+z^2-3y^2+x^2+y^2+z^2-3z^2}{(x^2+y^2+z^2)^{5/2}}$$

$$= 0$$

$$\iint_S \bar{F} \cdot d\bar{S} \quad , \text{where } \bar{F} = \langle z^2 x, \tfrac{1}{3}y^3 + \tan z, x^2 z + y^2 \rangle$$

S is top half of the sphere $x^2+y^2+z^2=1$



$\operatorname{div} \bar{F} = z^2+y^2+x^2$

Spherical!

$$\iiint \operatorname{div} \bar{F} \, dV = \iint_{S_2} - \iint_{S_1}$$

$$\iiint \operatorname{div} \bar{F} + \iint_{S_1} = \iint_{S_2}$$