§17.8

Stokes' Thm

$$\int_C \bar{F}\cdot d\bar{r} = \iint_S \text{curl}\,\bar{F}\cdot d\bar{S}$$

$$\boxed{\int_C \bar{F}\cdot\bar{T}\,ds} = \iint_S \text{curl}\,\bar{F}\cdot\bar{n}\,dS$$

$$\bar{n} = \frac{\bar{r}_u\times\bar{r}_v}{\cancel{|\bar{r}_u\times\bar{r}_v|}} \qquad dS = \cancel{|\bar{r}_u\times\bar{r}_v|}\,dA$$

$$F(b)-F(a) \qquad \int_a^b f'(x)\,dx$$

Special Case: S is flat, say, in xy-plane

$$\iint_S \text{curl}\,\bar{F}\cdot d\bar{S} = \iint_S (\text{curl}\,\bar{F}\cdot\bar{k})\,dA$$

$$\langle P, Q, 0\rangle = \bar{F} \qquad = \iint_S (Q_x - P_y)\,dA \quad \text{Green's}$$

$\boxed{E1}$ $\int_C \bar{F}\cdot d\bar{r}$, $\bar{F} = \langle -y^2, x, z^2\rangle$ and

C is the curve of intersection between

$y+z=2$ and the cylinder $x^2+y^2=1$

See Fig 3. Do we really want to parametrize C & do this line integral?

Stokes says "You don't have to."

$$\int_C \bar{F}\cdot d\bar{r} = \iint_S (\text{curl}\,\bar{F})\cdot d\bar{S} \qquad d\bar{S} = \bar{n}\,dS$$

$$\text{curl}\,\bar{F} = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\rangle \times \langle -y^2, x, z^2\rangle = \langle 0, 0, 1+2y\rangle$$

$y+z=2$

$0+y+z=2$

A normal to this plane is

$\langle 0,1,1\rangle$

unit normal:

$\frac{1}{\sqrt{2}}\langle 0,1,1\rangle = \bar{n}$.

$\bar{r} = \langle x, y, 2-y\rangle$

$\bar{r}_x = \langle 1,0,0\rangle$

$\times\ \bar{r}_y = \langle 0,1,-1\rangle$

$\langle 0,1,1\rangle$

Book just goes straight to here:

$$\iint_S \text{curl}\,\bar{F}\cdot \bar{r}_x\times\bar{r}_y\,dA = \iint_S \langle 0,0,1+2y\rangle\cdot\langle 0,1,1\rangle\,dA$$

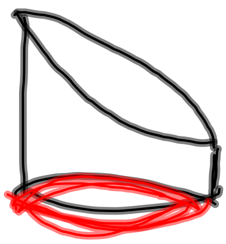
My way: $$\iint_S \langle 0,0,1+2y\rangle\cdot\frac{1}{\sqrt{2}}\langle 0,1,1\rangle\,dS \qquad |\bar{r}_x\times\bar{r}_y|$$

$$= \iint_S \langle 0,0,1+2y\rangle\cdot\langle 0,1,1\rangle\left(\frac{1}{\sqrt{2}}\right)\sqrt{2}\,dA$$

$$= \iint_S \langle 0,0,1+2y\rangle \cdot \langle 0,1,1\rangle \left(\frac{1}{\sqrt{2}}\right)\sqrt{2}\, dA$$

$$= \iint_S (1+2y)\, dA$$

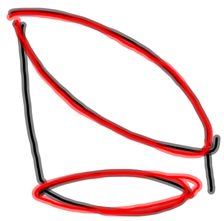
and in the xy-plane, it projects to a circle. use polar coords



$$\int_0^{2\pi}\int_0^1 (1+2r\sin\theta)\, r\, dr\, d\theta$$

$$= \pi$$

Example 1 w/o Stokes, i.e. do the line integral around the boundary.



Cylindrical coordinates are nice

$x = \cos\theta$, $y = \sin\theta$, $z = z$

C: $\bar{r} = \langle \cos\theta, \sin\theta, z \rangle$ & we know $z = 2 - y$,

so $\bar{r} = \langle \cos\theta, \sin\theta, 2 - \sin\theta \rangle$

$\bar{r}'(\theta) = \langle -\sin\theta, \cos\theta, -\cos\theta \rangle$ & so

$\bar{F} \cdot d\bar{r} = \bar{F}(\theta) \cdot \bar{r}'(\theta)\, d\theta =$

$\bar{F} = \langle -y^2, x, z^2 \rangle = \langle -\sin^2\theta, \cos\theta, (2-\sin\theta)^2 \rangle$

So $\bar{F}$

$$\int_C \bar{F} \cdot d\bar{r} = \int_C \langle -\sin^2\theta, \cos\theta, (2-\sin\theta)^2 \rangle \cdot \langle -\sin\theta, \cos\theta, -\cos\theta \rangle \, d\theta$$

$$\int_0^{2\cdot\pi} \left(\sin(t)^3 + \cos(t)^2 - 4\cos(t) + 4\cos(t)\sin(t) - \cos(t)\sin(t)^2\right) dt$$

$= \ldots\ldots\ldots = \pi$

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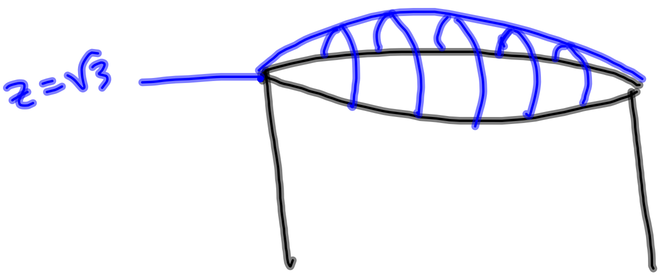
Example 2 goes the other way:

Evaluate $\iint_S \text{curl}\,\bar{F}\cdot\bar{n}\,dS$ by computing the line integral around $C = \partial S$



Stokes says

$\int_C \bar{F}\cdot d\bar{r}$

$\bar{r}$ is the circle $x^2+y^2=1$ at the top of this.

$x^2+y^2+z^2=4$

$x^2+y^2=1$

$1+z^2=4$

$z^2=3$

$z=\pm\sqrt{3}$

$\bar{r}=\langle\cos\theta, \sin\theta, \sqrt{3}\rangle$

$\bar{r}'=\langle-\sin\theta, \cos\theta, 0\rangle$

$\bar{F}\cdot\bar{r}'$

$=\langle xz, yz, xy\rangle\cdot\bar{r}'$

$=\langle\sqrt{3}\cos\theta, \sqrt{3}\sin\theta, \sin\theta\cos\theta\rangle$

$\cdot\langle-\sin\theta, \cos\theta, 0\rangle$

$=-\sqrt{3}\sin\theta\cos\theta+\sqrt{3}\sin\theta\cos\theta = 0 \Rightarrow$

$\int_C \bar{F}\cdot d\bar{r} = 0.$

Re-work Example 2 using just the top of the cylinder.

$\text{curl}\,\bar{F} = \langle x-y, -(y-x), 0\rangle$ and

$\bar{r} = \langle x, y, \sqrt{3}\rangle$

$\bar{r}_x = \langle 1, 0, 0\rangle$

$\times \quad \bar{r}_y = \langle 0, 1, 0\rangle$

$\langle 0, 0, 1\rangle$

$$\iint_{S'} \text{curl}\,\bar{F} \cdot d\bar{S} = \iint_{S} \langle x-y, -(y-x), 0\rangle \cdot \langle 0, 0, 1\rangle \, dS$$

$$= \iint_{S} 0 \, dS = 0$$

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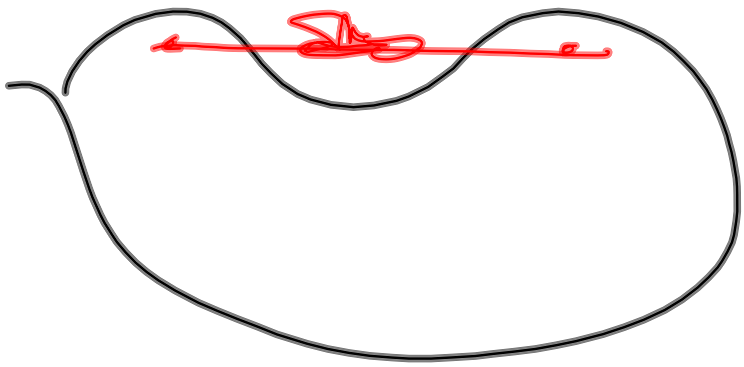
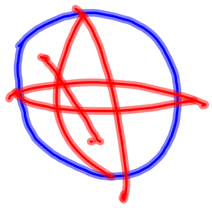
Divergence Thm:

$$\iint_S \bar{F} \cdot d\bar{S} = \iiint_E \operatorname{div} \bar{F} \, dV$$

Simple Solids: Think, basically, convex.





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