§ 17.7

Arc length led to Line Integrals ✓

Now -

Surface area leads to surface integrals ✓

$\bar{r} = \langle x(t), y(t) \rangle$

$$\text{Arc Length} = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\, dt = \int_C ds$$

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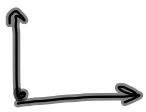
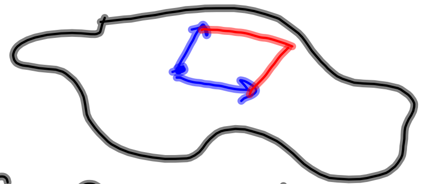
Line integral

$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t)) \sqrt{x'^2(t) + y'^2(t)}\, dt$$

$$= \int_a^b f(\bar{r}(t))\, |\bar{r}'(t)|\, dt$$





Surface Area

$$\iint_D |\bar{r}_u \times \bar{r}_v|\, dA$$

$z = f(x,y)$

17.6.9 $$\iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA$$

$\bar{r}(x,y) = \langle x, y, f(x,y) \rangle$

Surface Integral

$$\iint_S f(x,y,z)\, dS$$

$$\iint f(\bar{r}(u,v))\, |\bar{r}_u \times \bar{r}_v|\, dA$$

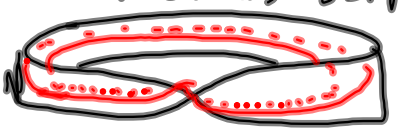
$\boxed{E1}$ $\iint_S x^2 \, dS$ where S is unit sphere

Mass of a sheet of copper with unit density function is $\rho(x,y,z)$, then total mass is $m = \iint_S \rho(x,y,z) \, dS$

Oriented Surfaces

Möebius Strip.

Can't be oriented



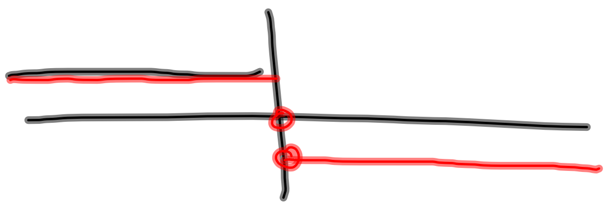
Klein Bottle

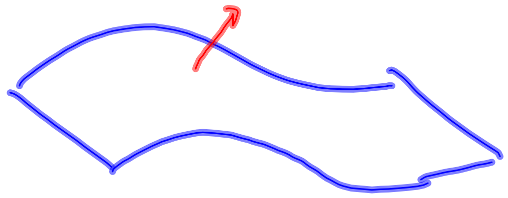
KLEIN

Smooth Orientable

1122 If $\bar{n}(x,y,z)$ can be chosen so that it varies continuously, then the surface has cnt^s partials (smoothness) & is an oriented surface.

$x^2\sin(\frac{1}{x})$





A "natural" unit normal to $\bar{r}(u,v)$ is $\bar{n} = \dfrac{\bar{r}_u \times \bar{r}_v}{|\bar{r}_u \times \bar{r}_v|}$ $\boxed{5}$

Closed Surface $\bar{n}$ points outward.

On the sphere, the outward normal is in the same direction as the position vector

$$x^2+y^2+z^2=\rho^2$$

$$\bar{n} = \frac{1}{\rho}\langle x,y,z\rangle$$

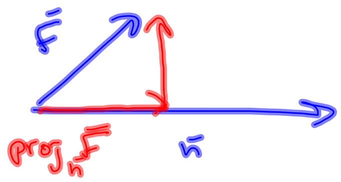
$$\iint_S \bar{F}\cdot d\bar{S} = \iint_S \bar{F}\cdot\bar{n}\,dS$$

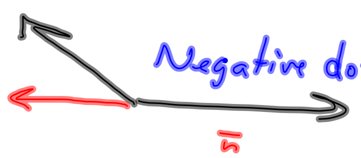
Size & direction (Differential change (vector))

Direction

differential change (scalar)

$\bar{F}\cdot\bar{n}$ is normal component of $\bar{F}$. It's related to $\text{proj}_{\bar{n}}\bar{F}$ $(\bar{F}\cdot\bar{n})$





Negative dot product.

$$\iint \bar{F} \cdot d\vec{S} = \iint_{S} \bar{F} \cdot \bar{n}\, dS$$

$$= \iint_{S} \bar{F} \cdot \frac{\bar{r}_u \times \bar{r}_v}{|\bar{r}_u \times \bar{r}_v|}\, dS$$

Think Jacobian.

$$= \iint_{S} \bar{F} \cdot \frac{\bar{r}_u \times \bar{r}_v}{\cancel{|\bar{r}_u \times \bar{r}_v|}} \cancel{|\bar{r}_u \times \bar{r}_v|}\, dA$$

Goal is to write as $\int_a^b \int_c^d$

Read Stokes, §17.8, §17.9 is Divergence Theorem

Refresh on

$$\text{curl } \bar{F} = \nabla \times \bar{F} = \langle 0, 0, Q_x - P_y \rangle$$

when we view $\bar{F} = \langle P, Q \rangle$ as $\langle P, Q, 0 \rangle$. Then $\nabla \times \bar{F}$ makes sense.

Stokes Generalizes Green's Theorem.