17.4 #14

$\bar{F} = \langle y - \ln(x^2+y^2), 2\tan^{-1}(\frac{y}{x}) \rangle$

C' is circle $(x-2)^2 + (y-5)^2 = 1$

$$\int_{C'} \bar{F} \cdot dr = \int_{C'} (y - \ln(x^2+y^2))\,dx + \underline{(2\tan^{-1}(\tfrac{y}{x}))}\,dy$$

$$= \iint_{D} \left[\frac{d}{dx}\left(2\tan^{-1}\left(\frac{y}{x}\right)\right) - \frac{d}{dy}\left(y - \ln(x^2+y^2)\right)\right] dA$$

$$= \iint_{D} \left[2\left(\frac{-\frac{y}{x^2}}{1+\left(\frac{y}{x}\right)^2}\right) - \left(1 - \frac{2y}{x^2+y^2}\right)\right] dA$$

$$= \ldots = \iint_{D} (-1)\,dA = -\pi.$$

(25) Claim: $\operatorname{div}(f\bar{F}) = f \operatorname{div}\bar{F} + \bar{F}\cdot\nabla f$

$\bar{F} = \langle F_1, F_2, F_3 \rangle$

9b

$\boxed{\text{Pf}}$ $\operatorname{div}(f\bar{F}) =$

$$\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle fF_1, fF_2, fF_3 \rangle$$

$$= \frac{\partial (fF_1)}{\partial x} + \frac{\partial (fF_2)}{\partial y} + \frac{\partial (fF_3)}{\partial z}$$

$$= f_x F_1 + fF_{1x} + f_y F_2 + fF_{2y} + f_z F_3 + fF_{3z}$$

$$= f_x F_1 + f_y F_2 + f_z F_3 +$$

$$fF_{1x} + fF_{2y} + fF_{2z}$$

$$= \nabla f \cdot \bar{F} + f \operatorname{div}\bar{F}$$

§17.6 Parametric Surfaces.

$\bar{r}(u,v)$ represents a surface defined "over" the uv-plane.

$\bar{r} = \langle x(u,v), y(u,v), z(u,v) \rangle$

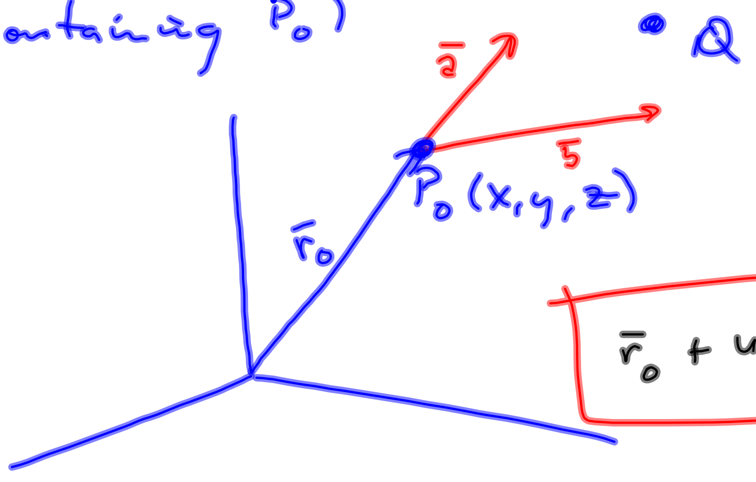
Grid Curves: Hold one parameter constant.

$\bar{r}(u_0, v)$ is a grid curve

It's a parametric curve, just like $\bar{r}(t)$ used to be.

Profile of the landscape

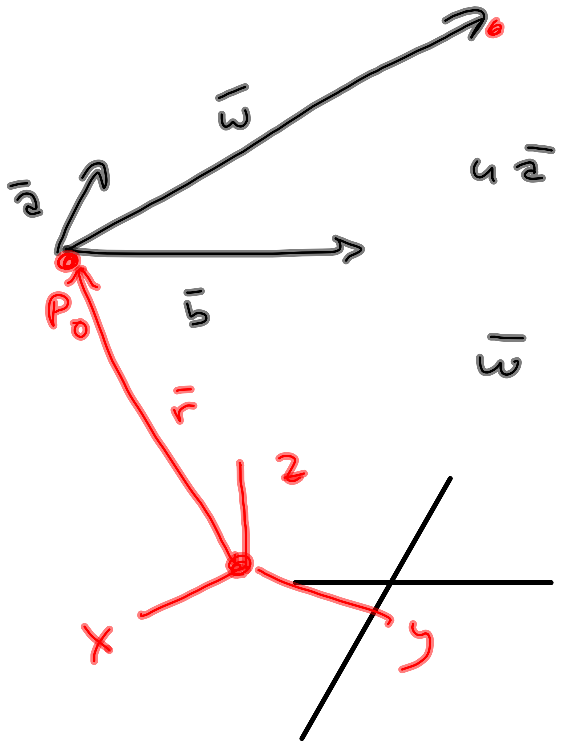
A plane in $\mathbb{R}^3$ thru P_0 that "contains" $\bar{a}, \bar{b}$. ($\bar{a}$ & $\bar{b}$ are actually in a plane thru the origin that's // to the plane containing P_0)



$$\boxed{\bar{r}_0 + u\bar{a} + v\bar{b}}$$

$\bar{r}(u,v) = r_0 + u\bar{a} + v\bar{b}$

$\bar{a}$ & $\bar{b}$ "span" the plane.



$$\bar{w} = u\bar{a} + v\bar{b}$$

E4 Sphere $x^2+y^2+z^2=\rho^2$

Spherical coords

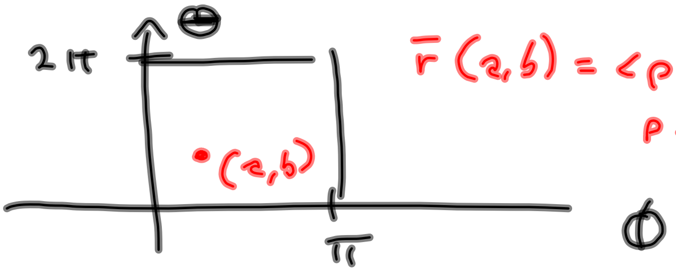
$x=\rho\sin\phi\cos\theta$

$y=\rho\sin\phi\sin\theta$

$z=\rho\cos\phi$

$\bar{r}(\phi,\theta)=\langle x(\phi,\theta), y(\phi,\theta), z(\phi,\theta)\rangle$

In the $\phi\theta$-plane, our surface is "over"



$\bar{r}(a,b)=\langle\rho\sin a\cos b, \rho\sin a\sin b, \rho\cos a\rangle$

Surface area:

A piece of $\bar{r}(u_0, v)$

Part of the grid curve $\bar{r}(u, v_0)$

|cross product| is

Area $|\bar{r}_v \Delta v \times \bar{r}_u \Delta u|$

$= |\bar{r}_v \times \bar{r}_u| \Delta v \Delta u$

$= |\bar{r}_v \times \bar{r}_u| \Delta A$

Think Jacobian

$$A(S) = \iint_{\mathcal{D}} |\bar{r}_u \times \bar{r}_v| \, dA$$

E10 Surface area of a sphere of radius a:

$\bar{r} = \langle a \sin\phi \cos\theta, a \sin\phi \sin\theta, a \cos\phi \rangle$

Surface area $|\bar{r}_\phi \times \bar{r}_\theta| = a^2 \sin\phi$

$$A(S) = \int_0^{2\pi} \int_0^{\pi} a^2 \sin\phi \, d\phi \, d\theta =$$

$$= a^2 \int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi \, d\phi = a^2 [2\pi][2] = 4\pi a^2$$

$$-\cos\phi \Big]_0^{\pi} = -\cos\pi - (-\cos 0)$$

$$= -(-1) - (-(+1))$$

$= 2$