$\boxed{T3}$ curl $\nabla f = \bar{0}$

<u>pf</u> $\nabla \times \nabla f = \nabla \times \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$

$$= \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle$$
$$\times \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \rangle$$

$$\langle \frac{\partial^2 f}{\partial y \partial z} - \frac{\partial^2 f}{\partial z \partial y}, -\frac{\partial^2 f}{\partial x \partial z} + \frac{\partial^2 f}{\partial z \partial x}, \frac{\partial^2 f}{\partial x \partial y} - \frac{\partial^2 f}{\partial y \partial x} \rangle$$

$= \bar{0}$ by Clairaut.

Bottom of Pg 1099

If $\bar{F}$ is conservative, then curl $\bar{F} = \bar{0}$

(T3, since ⤴ means $\exists f \ni \nabla f = \bar{F}$)

If $\bar{F}$ is defined on $\mathbb{R}$ and it's components have cnt's partials and curl $\bar{F} = \bar{0}$, then $\bar{F}$ is conservative.

Just keep your domain $\mathcal{D}$ away from spots where $\bar{F}$ doesn't have cnt's partials.

As long as there's no hole in my loop, things are AOK. So

$$\nabla \times \bar{F} = \bar{0} \Longleftrightarrow \bar{F} \text{ is conservative.}$$

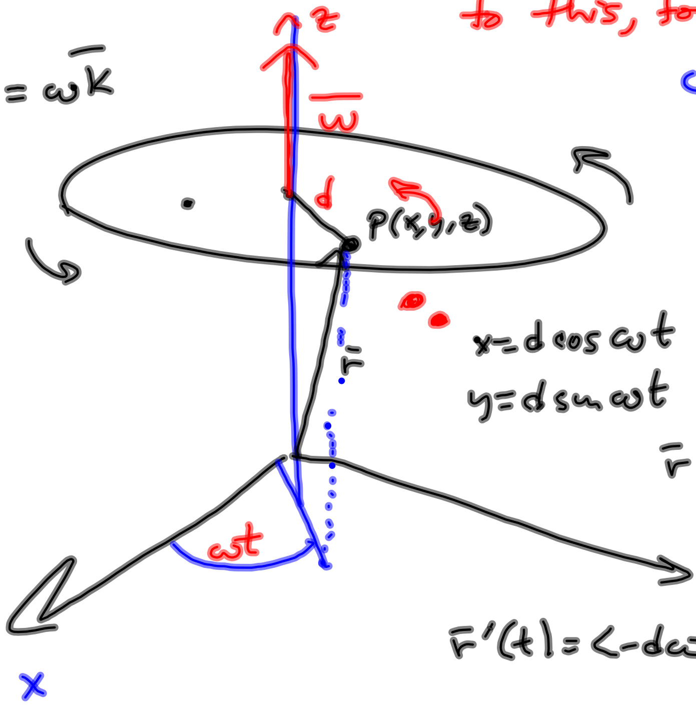
pretty much all the time.

Apr 24-12:14 PM

§17.5 #37

Come up with an alternate to this, for Bonus.

$\bar{w} = \omega \bar{k}$

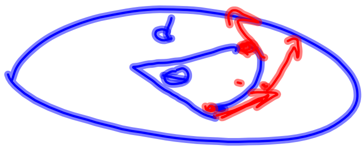


$x = d\cos\omega t$

$y = d\sin\omega t$

ω = angular speed $= \frac{d\theta}{dt}$

Arc length $= d\theta$



$\theta = \omega t$

$\bar{r} = \langle x, y, z \rangle$

$= \langle d\cos(\omega t), d\sin(\omega t), k \rangle$

$\bar{r}'(t) = \langle -d\omega\sin(\omega t), d\omega\cos(\omega t), 0 \rangle$

$= \langle -\omega y, \omega x, 0 \rangle = \bar{v}$

To me, that's how we get $\bar{v}$ = velocity vector.

(a) Show that $\bar{v} = \bar{w} \times \bar{r}$

$$\begin{array}{r} \langle 0, 0, \omega \rangle \\ \times \langle x, y, z \rangle \\ \hline \langle -\omega y, \omega x, 0 \rangle = \bar{v} \end{array}$$

$v = |\bar{v}|$

ω = angular speed $= \frac{v}{d}$

(b) $\bar{v} = \langle -\omega y, \omega x, 0 \rangle$

(c) $\text{curl}\,\bar{v} = 2\bar{w}$

$\omega \bar{k} = \langle 0, 0, \omega \rangle$

$$\begin{array}{r} \langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \rangle \\ \times \langle -\omega y, \omega x, 0 \rangle \\ \hline \langle 0, 0, \omega + \omega \rangle = 2\bar{w} \end{array}$$

Divergence: div $\bar{F} = \nabla \cdot \bar{F}$

$\bar{F} = \langle P, Q, R \rangle = \text{div}\,\bar{F} = P_x + Q_y + R_z$

$= \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$

a tendency for expansion

Flow in = flow out? div $\bar{F} = 0$

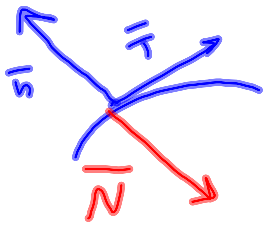
$\boxed{\text{T 11}}$ $\bar{F} = \langle P, Q, R \rangle$, P, Q, R have cts 2^{nd} order partials $\Rightarrow$ div curl $\bar{F} = 0$

$\boxed{\text{Pf}}$ Clairaut.

Recall $\bar{T} = \dfrac{\bar{r}'(t)}{|\bar{r}'(t)|}$ and <u>C14 stuff</u>

$$\boxed{\bar{N} = \frac{\bar{T}'(t)}{|\bar{T}'(t)|}}$$ is the Inward Unit Normal

$\bar{B} = \bar{T} \times \bar{N}$



check this out:

$\bar{n} = \dfrac{1}{|\bar{r}'(t)|} \langle y'(t), -x'(t) \rangle$ is the <u>Outward</u> Normal.

$\bar{n} \cdot \bar{T} = 0$

$\bar{N}$ from §14.3, then is $-\bar{n} = \dfrac{1}{|\bar{r}'(t)|} \langle -y'(t), x'(t) \rangle$

Vector form of Green's

$$\boxed{\int_C \bar{F} \cdot \bar{n}\, ds = \iint_D \operatorname{div} \bar{F}\, dA}$$

<u>Do read #38</u>

#33 Use Green's Thm Eq'n $\boxed{13}$ form to prove

$$\iint_D f\, \nabla^2 g\, dA = \int_C f(\nabla g) \cdot \bar{n}\, ds - \iint_D \nabla f \cdot \nabla g\, dA$$

$\nabla g \cdot \bar{n} = D_{\bar{n}}\, g$ is the Directional derivative in direction of $\bar{n}$, called the "normal derivative".

Identity in #25:

$$\int_C f(\nabla g)\cdot \bar{n}\, ds = \iint_D \left(\operatorname{div}(f \operatorname{div} \nabla g) + \nabla g \cdot \nabla f\right) dA$$

#35 Let $f(x,y)=1$. Then $\nabla f = 0$ & #33 gives it.