§17.4 Green's Theorem

Minimize Urgent Matters
by serving
Priorities

C is simple, positively oriented, closed curve in the plane.
$\mathcal{D}$ is an open domain, bdd by $C = \partial\mathcal{D}$

If P & Q have cont^{s} partials on an open region containing $\mathcal{D}$, then

$$\int_C P\,dx + Q\,dy = \iint_{\mathcal{D}} (Q_x - P_y)\,dA$$

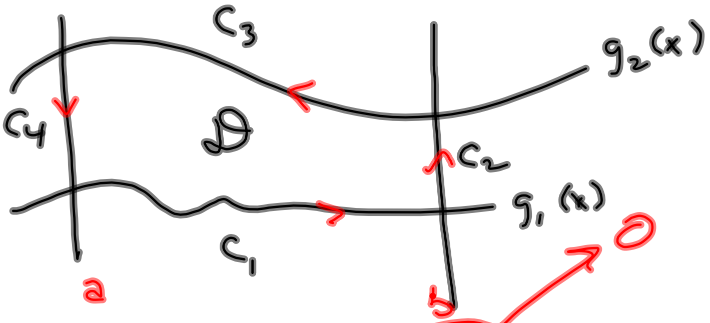
FTC II $\int_a^b F'(x)\,dx = F(b) - F(a)$

$I = [a,b]$
$\partial I = \{a,b\}$
$\partial B_2 = \partial\{(x,y) \mid x^2+y^2 \le r^2\} =$
$= \{(x,y) \mid x^2+y^2 = r^2\}$
$\partial B_3 = \{(x,y,z) \mid x^2+y^2+z^2 = r^2\}$

$$\int_{C} P\,dx + Q\,dy = \iint_{D} (Q_x - P_y)\,dA$$

I'll prove this piece!

$$\int_{C} P\,dx = -\iint_{D} P_y\,dA \quad , \text{ for a Type I } D \text{ region.}$$



$$\int_{C} P\,dx = \int_{C_1} P\,dx + \int_{C_2} P\,dx + \int_{C_3} P\,dx + \int_{C_4} P\,dx$$

0 $\quad$ 0

$$= \int_a^b P(x, g_1(x))\,dx + \int_b^a P(x, g_2(x))\,dx$$

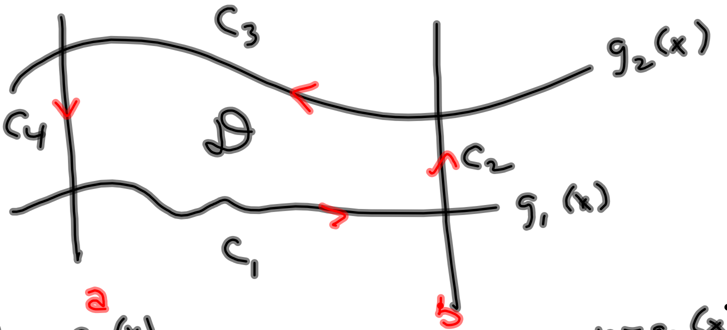
Let x be the parameter

$P(x(t), y(t))$
$= P(x, g_1(x))$

$$= \int_a^b \big(P(x, g_1(x)) - P(x, g_2(x))\big)\,dx$$

$$= -\int_a^b \big[P(x, g_2(x)) - P(x, g_1(x))\big]\,dx = \int_{C} P\,dx$$

$$= -\int_a^b \left[P(x, g_2(x)) - P(x, g_1(x))\right] dx = \int_C P\,dx$$



$$\iint_{\mathcal{D}} P_y \, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} P_y \, dy \, dx = \int_a^b \Big[P(x,y)\Big]_{y=g_1(x)}^{y=g_2(x)} dx$$

$$= \int_a^b \left[P(x, g_2(x)) - P(x, g_1(x))\right] dx = \iint_{\mathcal{D}} P_y \, dA$$

$$= -\int_C P\,dx$$

$\boxed{E2}$ $\int_C (3y - e^{\sin x})\,dx + (7x + \sqrt{y^4+1})\,dy$

$x^2+y^2 \le 9$ $= D$; $P = 3y - e^{\sin x}$, $Q = 7x + \sqrt{y^4+1}$

$$\iint_D (7-3)\,dA$$

$$= \int_0^{2\pi}\int_0^3 4\,r\,dr\,d\theta = 4\int_0^{2\pi} d\theta \int_0^3 r\,dr$$

$$= 4\left[2\pi\right]\left[\frac{9}{2}\right] = 36\pi$$

§ $\iint_{\mathcal{D}} (Q_x - P_y)\,dA$ is tough, but you know $P = Q = 0$ on $\partial \mathcal{D}$.

Green says $\iint_{\mathcal{D}} (Q_x - P_y)\,dA = 0$, too, then.

Another App. what's the area of $\mathcal{D}$?

$$\iint_{\mathcal{D}} 1\,dA$$

what if $\mathcal{D}$ is ugly?

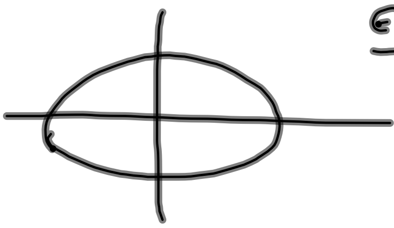
See Pg 1094.

Instead of $\iint$, find $P, Q \ni Q_x - P_y = 1$

E.g. $P \equiv 0$

$Q = x$

Then $$\iint_{\mathcal{D}} 1\,dA = \int_{\partial \mathcal{D} = C} 0\,dx + x\,dy$$



Example 3 Read See also, the other examples $\ni$ $Q_x - P_y = 1$

17.4 #s 3, 5, 7, 8, 12, 14, 27

17.5 #s 1, 3, 7, 13, 17, 19, 21, 22, 25, 35

§17.5 Curl & Divergence

$$\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$$

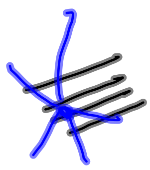
$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

$$\nabla^2 = \left\langle \frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2} \right\rangle$$

$$\bar{F} = \langle P, Q, R \rangle$$

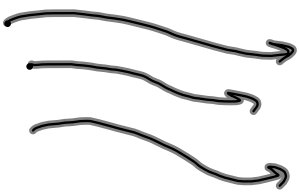
$\nabla \cdot \bar{F} =$ is scalar

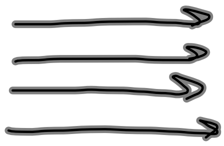
$$\nabla \times \bar{F} : \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}$$



$$= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\bar{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\bar{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\bar{k}$$

$$= \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle = \text{Curl } \bar{F}$$



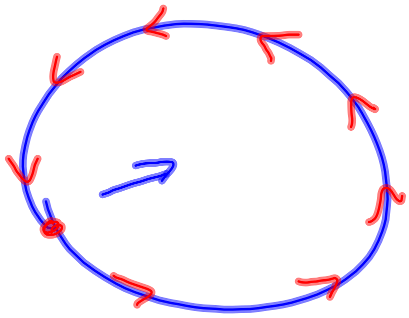


$\text{curl}(\nabla f) = \bar{0}$ ∇f

$\nabla \times \nabla f$

$\langle P, Q \rangle$

$Q_x = P_y$



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