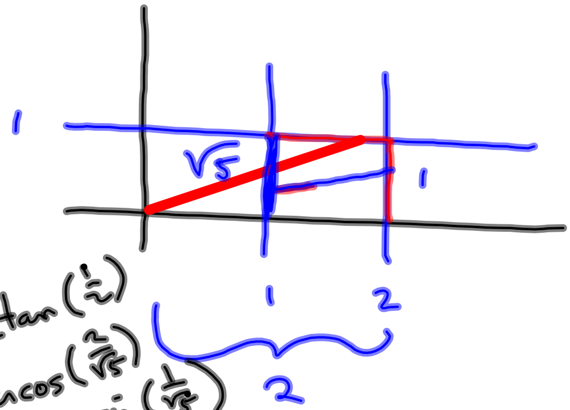
$$\int_0^1 \int_1^2 \frac{x}{x^2+y^2}\,dx\,dy$$

$x = r\cos\theta$

$y = r\sin\theta$

$dA = dx\,dy = r\,dr\,d\theta$



$\theta = \arcsin\left(\frac{1}{\sqrt{5}}\right)$

$x = 1 = r\cos\theta$

$\Rightarrow r = \sec\theta$

$x = 2 = r\cos\theta$

$r = 2\sec\theta$

$y = 1 = r\sin\theta$

$r = \csc\theta$

$\arctan\left(\frac{1}{2}\right)$

$\arccos\left(\frac{2}{\sqrt{5}}\right)$

$\arcsin\left(\frac{1}{\sqrt{5}}\right)$

$$\int_0^{\arcsin\left(\frac{1}{\sqrt{5}}\right)} \int_{\sec\theta}^{2\sec\theta} \frac{r\cos\theta}{r^2}\cdot r\,dr\,d\theta$$

$$+ \int_{\arcsin\left(\frac{1}{\sqrt{5}}\right)}^{\frac{\pi}{4}} \int_{\sec\theta}^{\csc\theta} \cos\theta\; r\,dr\,d\theta$$

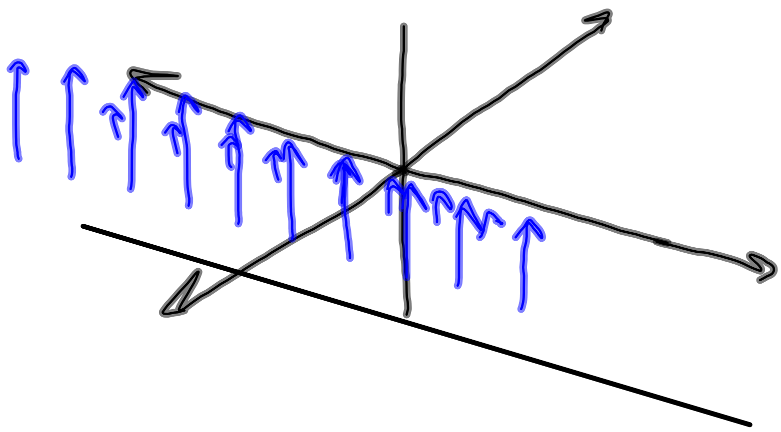
§17.1 Vector Fields

Recall sketching tangent fields?

$y' = f(x,y)$ Drawing arrows to represent tangent vector at (x,y)

$\bar{F} = \langle x(t), y(t) \rangle$

$\bar{F} = x\bar{k} = \langle 0, 0, x \rangle$



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§17.2 Line Integrals

§14.3 & 11.2

$$ds = \sqrt{x_t^2 + y_t^2}\, dt$$

$$\bar{r}(t) = \langle x(t), y(t) \rangle$$

$$\int_C f(x,y)\, ds$$

C is a smooth curve

$$\bar{r}(t) = \langle x(t), y(t) \rangle$$

(4) $\int_C x \sin y \, ds$ C is segment from (0,3) to (4,6)

$$\bar{r}(t) = (1-t)\langle 0,3\rangle + t\langle 4,6\rangle \qquad 0 \le t \le 1$$

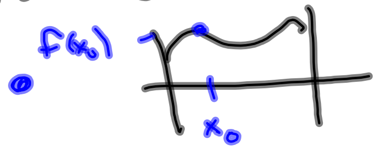
$$= \langle 0, 3-3t\rangle + \langle 4t, 6t\rangle$$

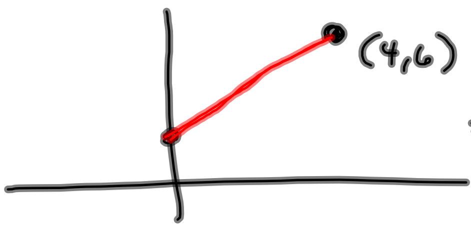
$$= \langle 4t, 3t+3\rangle = \bar{r}$$

$$\left.\begin{aligned} x &= x(t) = 4t \\ y &= 3t+3 \end{aligned}\right\}$$ when we're moving from (0,3) to (4,6) along the line segment C

$$\int_0^1 4t \sin(3t+3)\, 5\, dt$$

$$ds = \sqrt{\cancel{x_t^2} + \cancel{y_t^2}}\; dt = \sqrt{4^2+3^2} = \sqrt{25} = 5$$





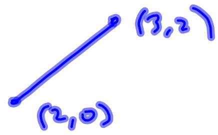
$$f(4,6) = 4 \sin 6$$

#7 $\boxed{\int_{C_2} xy\,dx + (x-y)\,dy}$ where C_2 is the line segment from $(2,0)$ to $(3,2)$

on C_2,

$x = x$, $y = 2x-4$

$dx = dx$ $dy = 2dx$



$m = \frac{2}{1} = 2$

$y = 2(x-2)+0$

$y = 2x-4$

This gives

$$\int_2^3 x(2x-4)\,dx + (x-(2x-4))\cdot 2\,dx$$

$$\int_{C_1} xy\,dx + \int_{C_1} (x-y)\,dy$$

C_1 = segment from $(0,0)$ to $(2,0)$

$y=0$

$0 \le x \le 2$

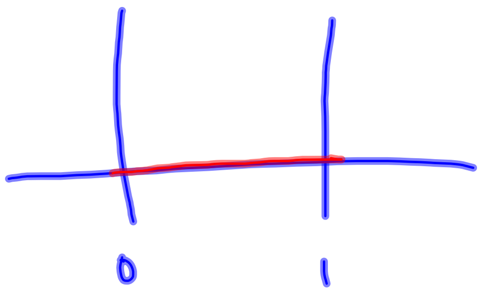
$x = x \quad y = 0$

$dx = dx \quad dy = 0\,dx$

This gives

$$\int_0^2 x\cdot 0\,dx + (x-0)\cdot 0\,dx = 0$$

$f(x)=0$



§17.3 #s 1, 7, 11, 12, 13, 19*, 21

Man, coming up with

$$ds = \sqrt{x'(t)^2 + y'(t)^2}\, dt$$ is

a giant pain in the whatever.

Having to describe C = curve over which we're integrating sucks.

$\boxed{T2}$ If C is a smooth curve, described by $\bar{r}(t) = \langle x(t), y(t) \rangle$ for $a \le t \le b$. If f is a differentiable function of 2 (or 3) variables with continuous gradient (∇f) on C, Then

$$\int_C \nabla f \cdot d\bar{r} = f(\bar{r}(b)) - f(\bar{r}(a))$$

This is the multi-dimensional version of $\int_a^b f'(x)\,dx = f(b) - f(a)$ FTC II.

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Recall

$\bar{F}$ is a conservative vector field if it's the gradient, ∇f, of some scalar function f, i.e., if

$$\bar{F} = \nabla f$$

Examples: See Gravity in §17.1

$$F = \frac{G m_1 m_2}{r^2} \quad \text{in 1-D.}$$

Read that stuff.

Is $\bar{F}$ conservative?

$$\bar{F} = \langle 2x-3y, -3x+4y-8 \rangle = \nabla f?$$

for some f?

$\nabla f = \langle f_x, f_y \rangle$

$f_x = 2x - 3y \implies f = ?$

$\int f_x \, dx = x^2 - 3yx + C$

$\implies \underline{f = x^2 - 3yx + g(y)}$

$f_y = -3x + g'(y) = -3x + 4y - 8.$

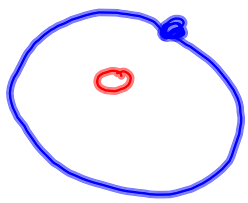
Apparently, $g'(y) = 4y - 8$, so

$\underline{g(y) = 2y^2 - 8y}$

$\implies f = x^2 - 3yx + 2y^2 - 8y$

$f_x = 2x - 3y$

$f_y = -3x + 4y - 8$



So $\vec{F}$ <u>is</u> conservative.

Sweet.

Conservative is good.

$\boxed{T3}$ If $\bar{F}$ is conservative on some region $\mathcal{D}$, then

$$\int_{C_1} \bar{F} \cdot d\bar{r} = \int_{C_2} \bar{F} \cdot d\bar{r}$$

for any paths C_1, C_2 contained in $\mathcal{D}$.

$\boxed{T4}$ $\S$ $\bar{F}$ is cnt^s V.F. on an open, connected domain $\mathcal{D}$. (1, 2)

If $\int_C \bar{F} \cdot d\bar{r}$ is independent of path, in $\mathcal{D}$, then $\underbrace{\bar{F} \text{ is conservative.}}_{\exists f \ni \nabla f = \bar{F}}$

Be careful to know which way the arrow points

implication

BEWARE THE Converse!

$\boxed{T5}$ If $\bar{F} = \langle P(x,y), Q(x,y) \rangle$ is conservative and P, Q have cnt^s 1^st partials, then

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

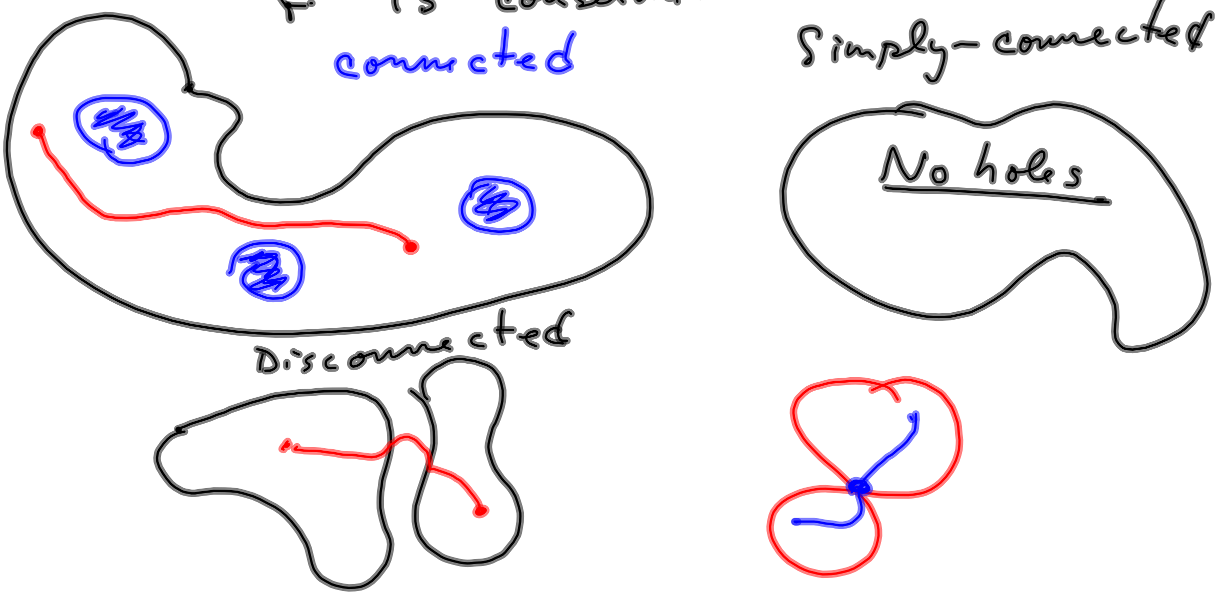
(If $\bar{F}$'s conservative, then

$$\left.\begin{aligned} P = f_x &\Rightarrow P_y = f_{xy} \\ Q = f_y &\Rightarrow Q_x = f_{yx} \end{aligned}\right\} \text{Clairaut's Thm.}$$

$\boxed{T6}$ A sort of converse

If $\bar{F} = \langle P, Q \rangle$ is defined on an open, simply-connected D and P & Q have cnt^s 1st order partials

Then, if $P_y = Q_x$, we know $\bar{F}$ is conservative.



Show that $\bar{F} = \langle \underline{2x-3y}, -3\underline{x+4y-8} \rangle$ is conservative:

$P_y = -3$

$Q_x = -3$ Done.

Connected? Simply Connected?

$\bar{F}$ is well-behaved on the entire plane $\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

This stuff only really comes in when $\bar{F}$ has continuity issues.

$$\bar{F} = \left\langle \frac{x}{x+y}, \frac{x}{2x-5} \right\rangle$$

Have to stay away from

$x = -y$ &

$x = \frac{5}{2}$