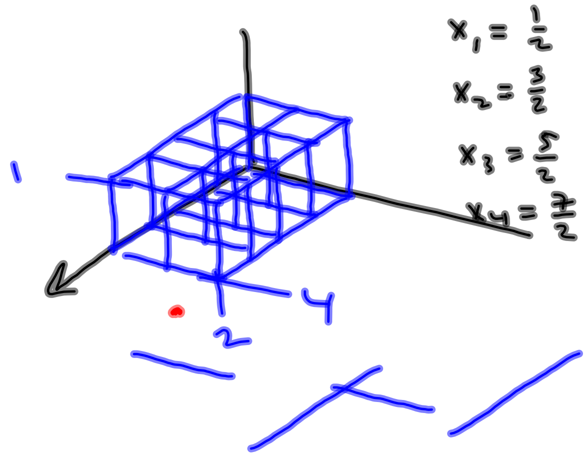
16.6 #26

$B = \{ (x,y,z) \mid 0 \le x \le 4, 0 \le y \le 2, 0 \le z \le 1 \}$



$x_1 = \frac{1}{2}$ $y_1 = \frac{1}{2}$ $z_1 = \frac{1}{2}$

$x_2 = \frac{3}{2}$ $y_2 = \frac{3}{2}$

$x_3 = \frac{5}{2}$

$x_4 = \frac{7}{2}$

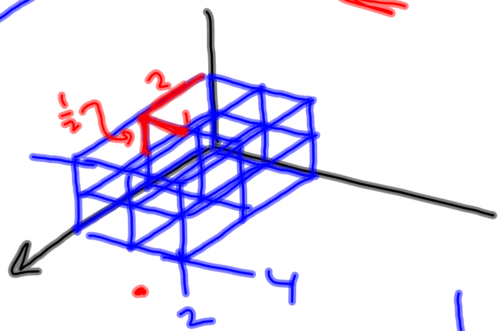
That's what I thought

$x_i = \frac{2i-1}{2}, \quad i=1,\dots,4$

$y_j = \frac{2j-1}{2}, \quad j=1,\dots,2$

$z_k = \frac{2k-1}{2}, \quad k=1$

$$\iiint_B \sin(xy^2z^3)\, dV$$



Book way

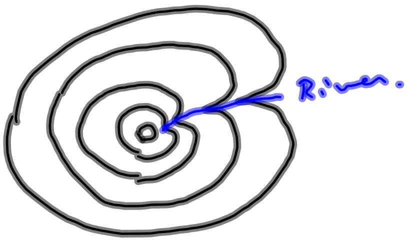
$x_1 = 1 \quad y_1 = \frac{1}{2} \quad z_1 = \frac{1}{4}$

$x_2 = 3 \quad y_2 = \frac{3}{2} \quad z_2 = \frac{3}{4}$

$\Delta V = 1$

$$\sum_{k=1}^{2} \sum_{j=1}^{2} \sum_{i=1}^{2} \sin\left((2i-1)\left(\frac{2j-1}{2}\right)^2\left(\frac{2k-1}{4}\right)^3\right) \cdot 1$$

16.1 A sketch? See Gradient?



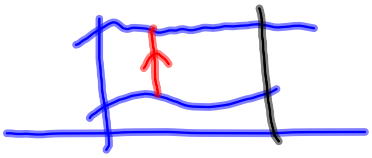
16.2 Fubini: Stuff Evaluate

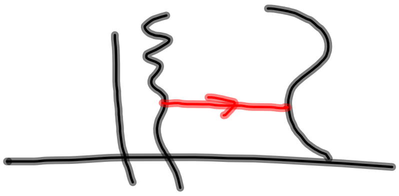
$\iint_R$ & $\iiint_E$ on rectangular domains.

$R = [a,b] \times [c,d]$

$E = [a,b] \times [c,d] \times [e,f]$

16.3 Type I, II



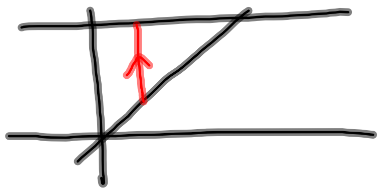


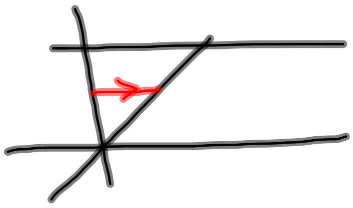
$\iint f\,dy\,dx$ $\qquad$ $\iint f\,dx\,dy$

Maybe a "do it I & II way".

#48

$$\int_0^1\int_x^1 e^{\frac{x}{y}}\,dy\,dx = \int_0^1\int_0^y e^{\frac{x}{y}}\,dx\,dy$$





$$= \int_0^1\int_0^y y e^{\frac{x}{y}}\cdot\frac{1}{y}\,dx\,dy = \int_0^1 \left[y e^{\frac{x}{y}}\right]_{0=x}^{y=x} dy$$

$u = \frac{x}{y}$ $\quad du = \frac{1}{y}dx$

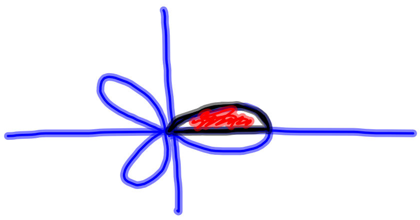
y is constant

$$= \int_0^1 [ye - y]\,dy$$

$$= \int_0^1 (e-1)y\,dy = \frac{1}{2}(e-1)\left[y^2\right]_0^1 = \frac{1}{2}(e-1)$$

16.4 Polar coordinates.

Area of one loop of $r = \cos(3\theta)$



old way:

$A = \frac{1}{2} r^2 \theta$

$A = \frac{1}{2} f(\theta)^2 \cdot \theta$

$\cos 3\theta = 0$ @

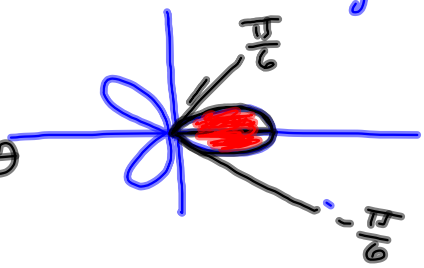
$3\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}$

$$A = 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{6}} \cos^2(3\theta)\, d\theta = \frac{\pi}{12}$$

16.4 way: Find the volume over the region of $f(r,\theta) = 1$

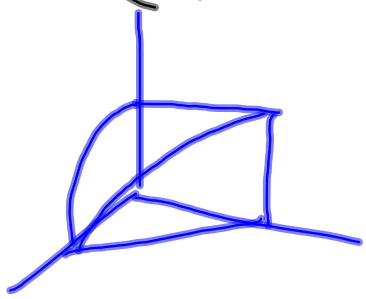
$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \int_0^{\cos(3\theta)} 1\, r\, dr\, d\theta$$



§16.6 write an integral like #12

Take-home: 6 ways to write a triple integral.

Something like #32

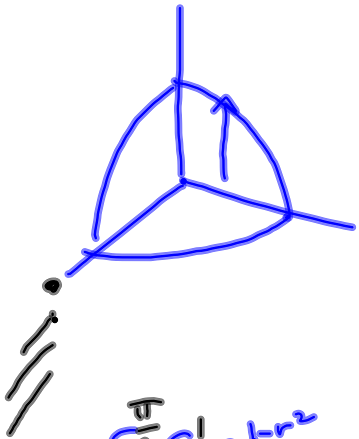


§16.7 cylindricals.

#18 $E = \{(x,y,z) \mid 0 \le z \le 1-x^2-y^2,\ x \ge 0, y \ge 0\}$

cylindricals $z = 1-(x^2+y^2)$

$= 1 - r^2$



$0 \le r \le 1$

$$\int_0^{\frac{\pi}{2}}\int_0^1\int_0^{1-r^2} f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

Setting one up: Totally within reason.

$x^2+y^2=r^2$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

§16.8 orange slices.

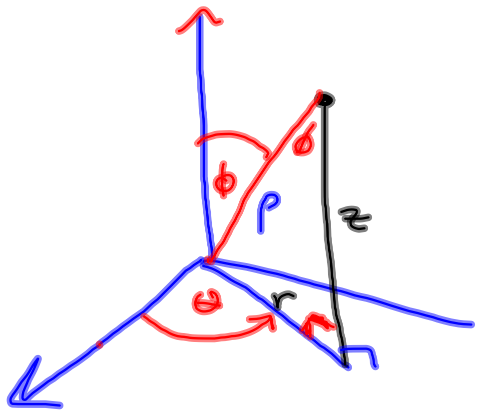
$(x, y, z) \longrightarrow (\rho, \theta, \phi)$

$\underline{\quad r \quad}$

$x = r\cos\theta$
$= \rho\sin\phi\underline{\cos\theta}$

$y = r\sin\theta$
$= \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$



When?

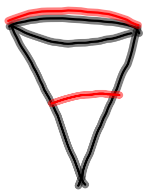
when E is spherical.

cones

$x^2+y^2+z^2$ in f

x^2+y^2 in f

#26 $\iiint_E xyz\, dV$ is a good one



$\frac{1}{20}\sin(1)$

$(2x)^2 - x^2 = 3x^2$

(14) §16.8 $\rho \le 2$, $\underline{\rho \le \csc\phi}$

$\rho \le \csc\phi$

$\rho\sin\phi \le 1$, so, when $\rho = 2$,

$\sin\phi \le \frac{1}{2}$ No help.

We know we're inside the sphere $\rho = 2$.

$\rho \le \csc\phi$

$\rho\sin\phi \le 1$ $\rho\sin\phi$

$\rho^2\sin^2\phi \le 1$ $\rho^2\sin^2\phi$

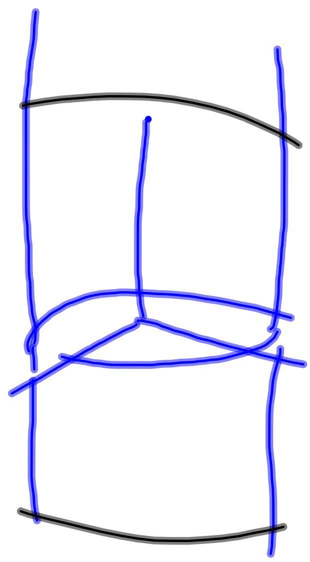
$\rho^2\sin^2\phi\underline{(\sin^2\theta + \cos^2\theta)} \le 1$

$\rho^2\sin^2\phi\underline{\sin^2\theta} + \rho^2\sin^2\phi\underline{\cos^2\theta} \le 1$

$y^2 + x^2 \le 1$

$x^2 + y^2 \le 1$

and $\rho \le 2$



$$\boxed{\iiint f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv}$$

$\left|\frac{\partial(x,y)}{\partial(u,v)}\right|$ is like a magnifying factor from one universe to the next.

It's how we go from $dA = dx\,dy$

to $dA = \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$

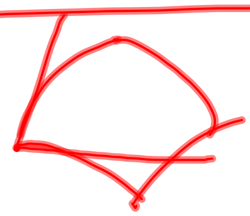
§16.9

Map one region to another

The connection between polar coords & 16.9's change-of-variables approach.

$$\iint f(r,\theta)\, r\, dr\, d\theta$$

16.9 $$\boxed{\iint f(x(r,\theta),y(r,\theta))\overbrace{\left|\frac{\partial(x,y)}{\partial(r,\theta)}\right|}^{r} dr\,d\theta}$$



$u = x-3y$
$v = x+y$ } solve for x & y

$\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$