§16.9 #12

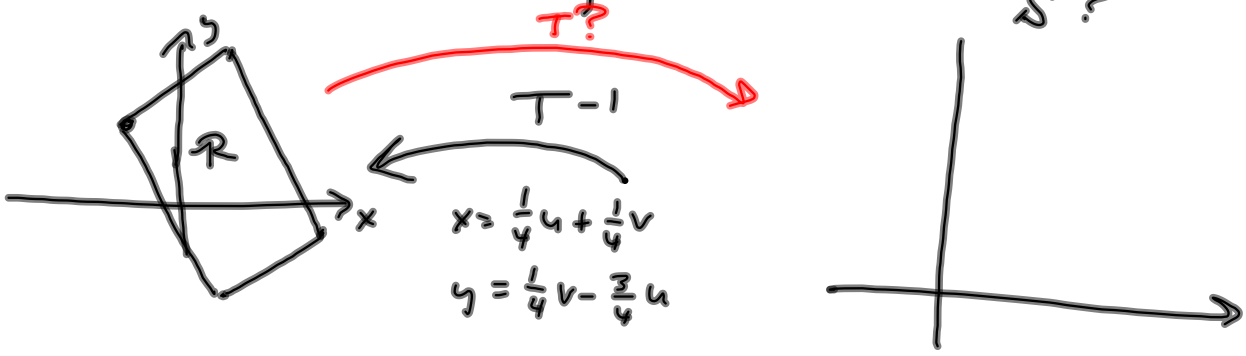
Given:

$$\iint_R (4x+8y)\,dA$$, where R is the parallelogram

(-1,3), (1,-3), (3,-1), (1,5)

$x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$ makes $\frac{\partial(x,y)}{\partial(u,v)}$ easy.

We evaluate the integral



Solve for u & v in terms of x & y.

$x = \frac{1}{4}u + \frac{1}{4}v$

$y = \frac{1}{4}v - \frac{3}{4}u$

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

$x + 2y = 7$

$-3x + 5y = 8$

$$\left[\begin{array}{cc|c} 1 & 2 & 7 \\ -3 & 5 & 8 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 11 & 29 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 7 \\ 0 & 1 & \frac{29}{11} \end{array}\right]$$

$3R1 + R2$ $\qquad \frac{1}{11}R2$

$\frac{77 - 58}{11} = \frac{19}{11}$

$$\sim \left[\begin{array}{cc|c} 1 & 0 & 7 - 2\left(\frac{29}{11}\right) \\ 0 & 1 & \frac{29}{11} \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & \frac{19}{11} \\ 0 & 1 & \frac{29}{11} \end{array}\right]$$

$-2R2 + R1$

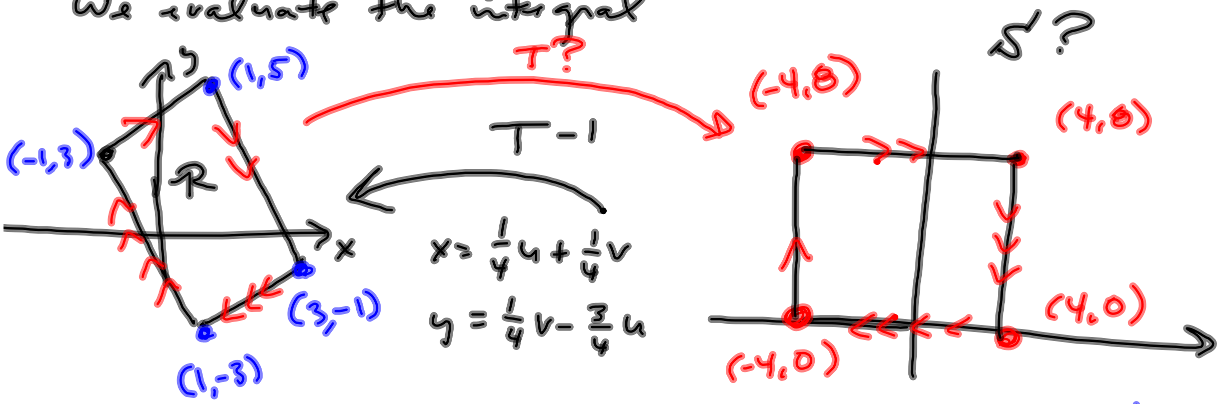
$$\left[\begin{array}{cc|c} \frac{1}{4} & \frac{1}{4} & x \\ -\frac{3}{4} & \frac{1}{4} & y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 4x \\ -3 & 1 & 4y \end{array}\right] \sim$$

$$\left[\begin{array}{cc|c} 1 & 1 & 4x \\ 0 & 4 & 12x+4y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & 4x \\ 0 & 1 & 3x+y \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & x-y \\ 0 & 1 & 3x+y \end{array}\right]$$

$u = x - y$

$v = 3x + y$

We evaluate the integral



$u = x - y$
$v = 3x + y$

This gives us T!

$T(-1,3) = (-4,0)$
$T(1,5) = (-4,8)$
$T(3,-1) = (4,8)$
$T(1,-3) = (4,0)$

we're sure it doesn't move around between endpoints, because R is bounded by straight lines and T involves linear stuff only

This gives $\int_{-4}^{4}\int_{0}^{8} (3v-5u)\cdot\frac{1}{4}\cdot dv\,du$

$4x+8y = 4\left(\frac{1}{4}u+\frac{1}{4}v\right) + 8\left(\frac{1}{4}v-\frac{3}{4}u\right)$
$= u+v+2v-6u$
$= -5u+3v$

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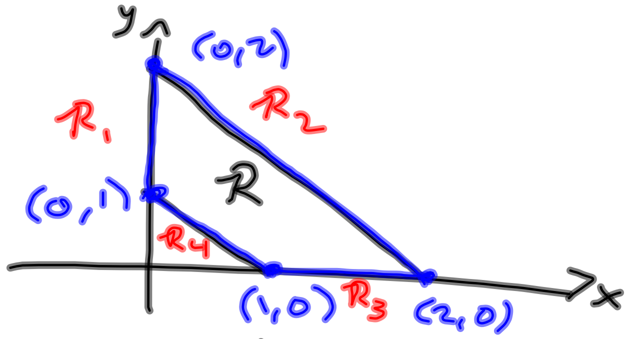
(21) $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$ R is trapezoid



Let $u = y-x$
$v = x+y$

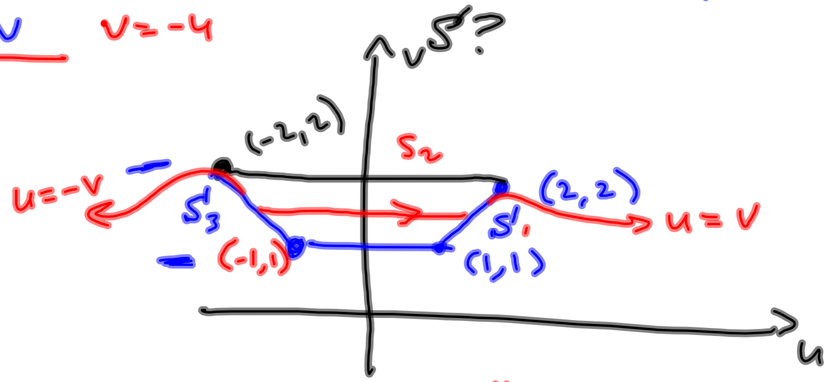
R_3: $y=0$ $1 \le x \le 2$

$u = -x$, $v = x$ } $u = -v$ $v=-4$

$1 \le x \le 2$

$-1 \ge u \ge -2$

$-2 \le u \le -1$



$$\int_1^2 \int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv$$

R_2: $y = -x+2$ $0 \le x \le 2$

$u = -x+2-x = -2x+2$

$v = x + (-x+2) = 2 = v$

$0 \le x \le 2$ $0 \ge -x \ge -2$

$0 \ge -2x \ge -4$

$2 \ge -2x+2 \ge -2$

$-2 \le u \le 2$

R_1: $x=0$, $1 \le y \le 2$

$u = y$, $1 \le y \le 2$

$v = y$ $1 \le u = v \le 2$

So, $u=v$

All that's missing now, is the Jacobian

Let $u = y - x$
$v = x + y$

$$\begin{aligned} x + y &= v \\ -x + y &= u \end{aligned} \implies \begin{aligned} x &= v - y \\ x &= y - u \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 1 & v \\ -1 & 1 & u \end{array}\right]$$

$y - u = v - y$

$2y = u + v$

$y = \frac{1}{2}(u+v) = \frac{1}{2}u + \frac{1}{2}v = y$

$$\left[\begin{array}{cc|c} 1 & 1 & v \\ 0 & 2 & v+u \end{array}\right]$$

So $x = v - \left(\frac{1}{2}(u+v)\right)$

$= v - \frac{1}{2}u - \frac{1}{2}v$

$x = \frac{1}{2}v - \frac{1}{2}u$

$$\left[\begin{array}{cc|c} 1 & 1 & v \\ 0 & 1 & \frac{1}{2}v + \frac{1}{2}u \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 0 & \frac{1}{2}v - \frac{1}{2}u \\ 0 & 1 & \frac{1}{2}v + \frac{1}{2}u \end{array}\right] \begin{array}{l} = x \\ = y \end{array}$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

Sweet

$$\int_1^2 \int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\, dv \quad \int_1^2 \int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \cdot \frac{1}{2} \cdot du\, dv$$

$$= \frac{3}{2}\sin(1)$$

(22) $\iint_R \sin(9x^2 + 4y^2)\, dA = \iint_D \sin(u^2+v^2) \cdot \frac{1}{6} \cdot dv\, dy$

$u = 3x,\ v = 2y$

$\frac{1}{4}$ of ellipse $\quad$ $\frac{1}{4}$ of circle

$$= \frac{1}{6} \int_0^{\frac{\pi}{2}} \int_0^1 \sin(r^2)\, r\, dr\, d\theta = \frac{\pi}{24}(1 - \cos(1))$$