¶16 Test Monday

§16.9 #s 1-4, 7, 8, 11, 19, 20

Find the Jacobian

$x = x(u,v,w) = \frac{u}{v}, \quad y = \frac{v}{w}, \quad z = \frac{w}{u}$

$$\begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} & 0 \\ 0 & \frac{1}{w} & -\frac{v}{w^2} \\ -\frac{w}{u^2} & 0 & \frac{1}{u} \end{vmatrix}$$

Expansion by minors

$$= \frac{1}{v}\left(\frac{1}{uw} - 0\right) - \left(-\frac{u}{v^2}\right)\left(0 - \left(\frac{wv}{u^2w^2}\right)\right) + 0\,(\text{——})$$

$$= \frac{1}{v}\begin{vmatrix} \frac{1}{w} & \frac{-v}{w^2} \\ 0 & \frac{1}{u} \end{vmatrix} - \left(-\frac{u}{v^2}\right)\begin{vmatrix} 0 & -\frac{v}{w^2} \\ -\frac{w}{u^2} & \frac{1}{u} \end{vmatrix} + 0\begin{vmatrix} 0 & \frac{1}{w} \\ -\frac{w}{u^2} & 0 \end{vmatrix}$$

$$= \frac{1}{uvw} - \frac{uvw}{u^2v^2w^2} = 0 \ !?$$

3 $x = e^{-r}\sin\theta$, $y = e^{r}\cos\theta$

Derivation/theory.

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} x_r & x_\theta \\ y_r & y_\theta \end{vmatrix} = \begin{vmatrix} x_r & y_r \\ x_\theta & y_\theta \end{vmatrix}$$

From Friday's Notes.

$$= \begin{vmatrix} -e^{-r}\sin\theta & e^{-r}\cos\theta \\ e^{r}\cos\theta & -e^{r}\sin\theta \end{vmatrix}$$

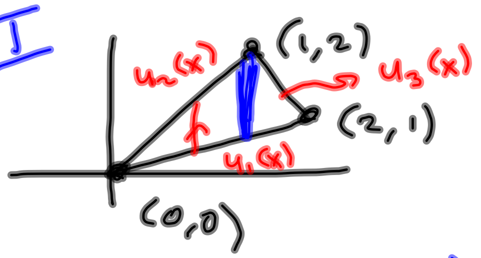
$$= (e^{-r}\sin\theta)(e^{r}\sin\theta) - (e^{r}\cos\theta)(e^{-r}\cos\theta)$$

$$= \boxed{\sin^2\theta \overset{\text{is fine}}{-} \cos^2\theta} = -(\cos^2\theta - \sin^2\theta) = \boxed{-\cos(2\theta)}$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

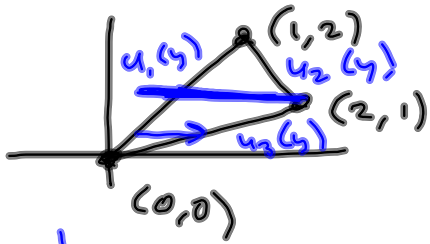
#11 gives us a case where we want to transform the region, because in this case T I and T II methods both require 2 integrals

I



$$\int_0^1 \int_{u_1(x)}^{u_2(x)} f\,dy\,dx + \int_1^2 \int_{u_1(x)}^{u_3(x)} f\,dy\,dx$$

II



$$\int_0^1 \int_{u_1(y)}^{u_3(y)} f\,dx\,dy + \int_1^2 \int_{u_1(y)}^{u_2(y)} f\,dx\,dy$$

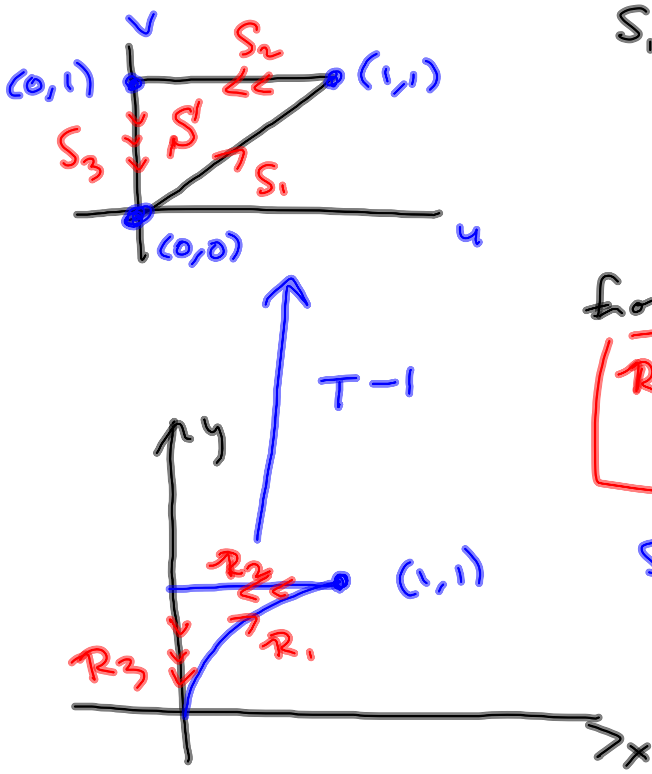
#9 Find $T(S')$, where

S' is triangle w/ vertices $(0,0)$, $(1,1)$, $(0,1)$

$T(u,v) = (u^2, v)$

$x = u^2$ $\quad$ $x = x(u,v) = u^2$

$y = v$ $\quad$ $y = y(u,v) = v^2$



S_1: $0 \le u \le 1$
$0 \le v \le 1$ $\quad$ $v = u$

$x = u^2$ $\quad$ $y = v = u$

So $x = y^2$

from $(0,0)$ to $(1,1)$ becomes

R_1 $(0,0)$ to $(1,1)$ on the curve $x = y^2$

S_2: $v = 1$, $0 \le u \le 1$

$x = u^2$, $y = v = 1$

$y = 1$ is it.

From $(1,1)$ to $(0,1)$

R_2 $(1,1)$ to $(0,1)$ on the curve $y = 1$

S_3: $u = 0$, $0 \le v \le 1$

$x = u^2 = 0$ $\quad$ $y = v$

From $(0,1)$ to $(0,0)$

R_3 $(0,1)$ to $(0,0)$ on the curve $x = 0$

$$\iint_R (4x+8y)\,dA \text{ , where } R \text{ is the parallelogram}$$

$(-1,3),\ (1,-3),\ (3,-1),\ (1,5)$

SIDEWAYS, DUMMY.

$x = \frac{1}{4}(u+v)$, $y = \frac{1}{4}(v-3u)$

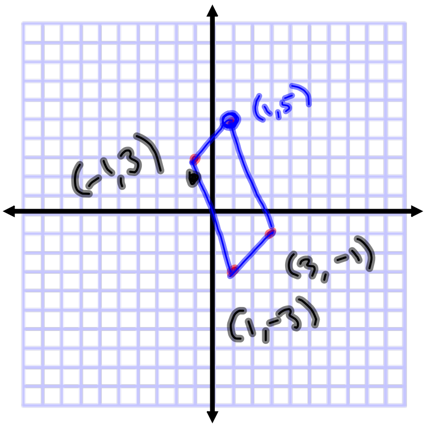
$u = x-y$ &
$v = 3x+y$

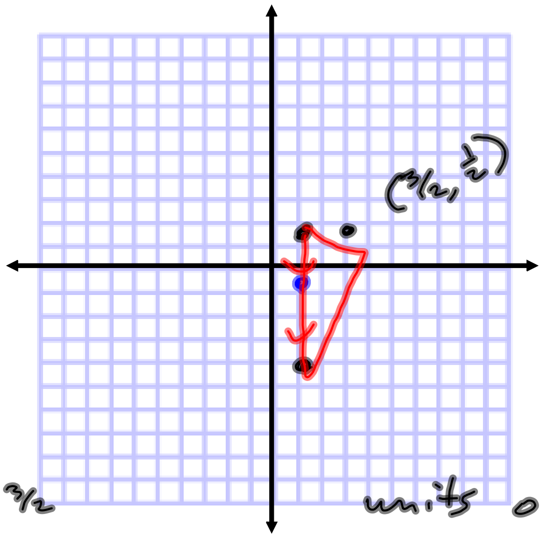
$$\iint_S \left((u+v) + 2(v-3u)\right)\frac{1}{4}\,dA$$

$dA \to dudv$?
$dvdu$?

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{vmatrix} = \frac{1}{16} + \frac{3}{16} = \frac{1}{4}$$

$(-1,3),\ (1,-3),\ (3,-1),\ (1,5)$





units of $\frac{1}{2}$

$x = \frac{1}{4}(u+v)$ $\quad x = 3/2$

$y = \frac{1}{4}(v-3u)$ $\quad y = \frac{1}{2}$

$\frac{1}{4}(-1+3) = \frac{1}{2}$

$\frac{1}{4}(3-3(1)) = 0$

$(1,-3)$

$x = \frac{1}{4}(1-3) = \frac{1}{2}$

$y = \frac{1}{4}(-3-3(1)) = -\frac{3}{2}$

$(3,-1)$

$x = \frac{1}{4}(3-1) = \frac{1}{4}(2) = \frac{1}{2}$

$y = \frac{1}{4}(3+3) = \frac{3}{2}$