$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\, dy\, dz\, dx$$

16.6 #34

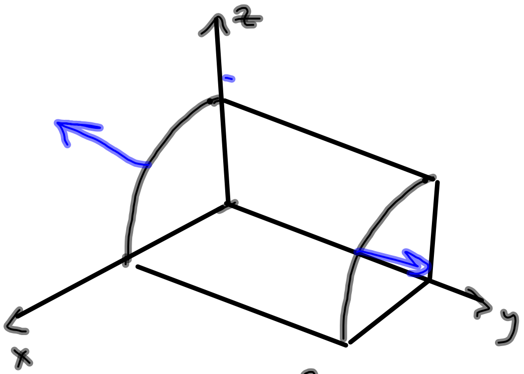
Submit
16.6 #32
Friday

$0 \le y \le 1-x$

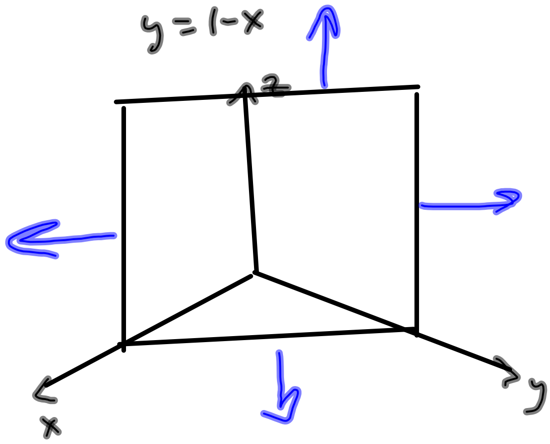
$0 \le z \le 1-x^2$

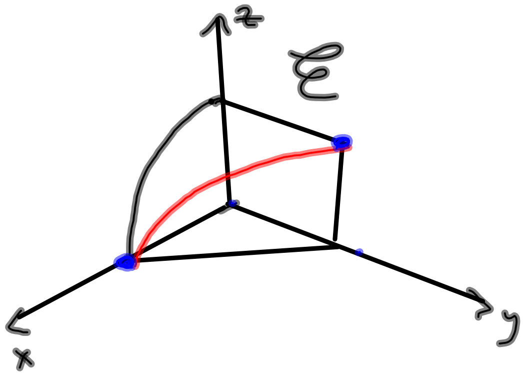
$0 \le x \le 1$

$z = 1-x^2$

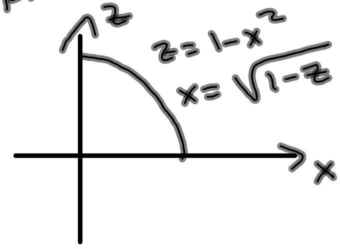


$y = 1-x$

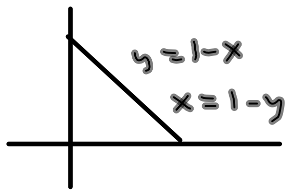


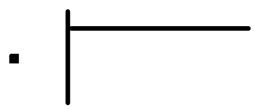


xz-plane

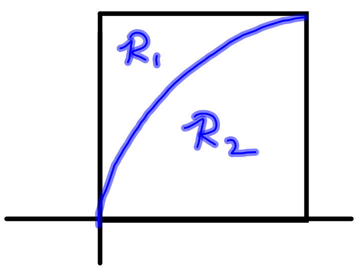


xy-plane





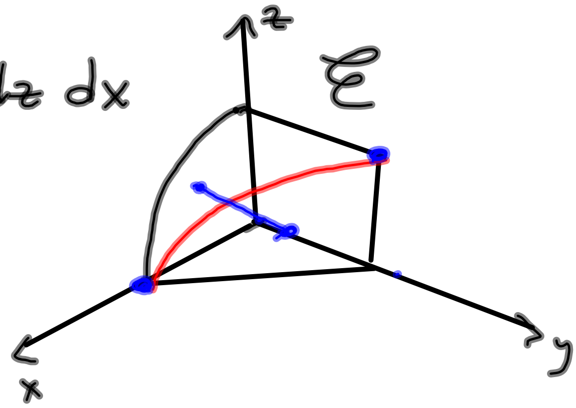
yz-plane



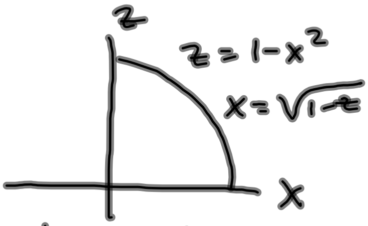
$\iiint f\, dx\, dy\, dz$ &

$\iint f\, dx\, dz\, dy$ will be TOUGH.

$$\int_0^1\int_0^{1-x^2}\int_0^{1-x} f(x,y,z)\,dy\,dz\,dx$$

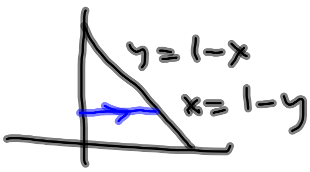


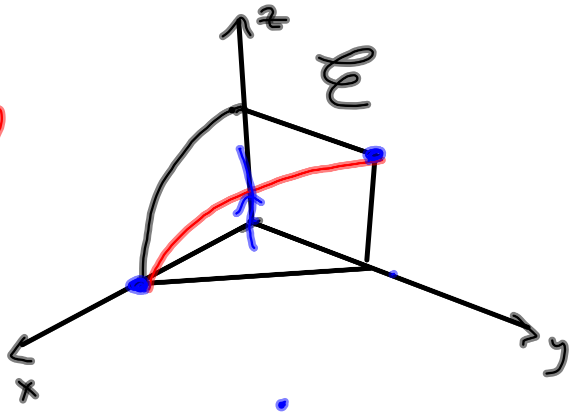
$$=\int_0^1\int_{x=0}^{\sqrt{1-z}}\int_0^{1-x} f\,dy\,dx\,dz$$



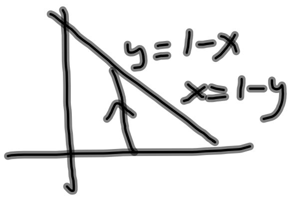
Now, integrate wrt z, first

$$\int_{0=y}^{y=1}\int_{x=0}^{x=1-y}\int_0^{1-x^2} f\,dz\,dx\,dy$$

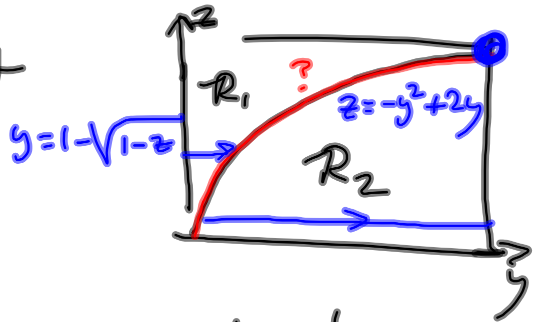




$$\int_{x=0}^{x=1}\int_{y=0}^{y=1-x}\int_{z=0}^{1-x^2} f\,dz\,dy\,dx$$



Now for the tricky part

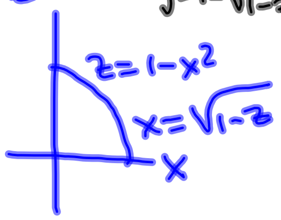


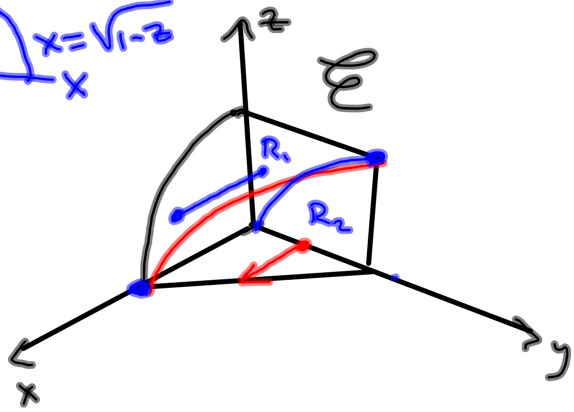
R_1

$$\int_{z=0}^{1}\int_{y=0}^{y=1-\sqrt{1-z}}\int_{x=0}^{x=\sqrt{1-z}} f\,dx\,dy\,dz \;+$$

R_2

$$\int_{z=0}^{z=1}\int_{y=1-\sqrt{1-z}}^{y=1}\int_{x=0}^{x=1-y} f\,dx\,dy\,dz$$





The boundary between R_1 & R_2 is the projection of the intersection of $z=1-x^2$ &

$$y = 1-x$$

$$\Rightarrow x = 1-y$$

$$\Rightarrow z = \underline{1-(1-y)^2}$$

$$z = 1-(y^2-2y+1)$$

$$= 1-y^2+2y-1$$

$$z = -y^2+2y$$

Also

$$1-(y-1)^2 = z$$

$$-(y-1)^2 = z-1$$

$$(y-1)^2 = 1-z$$

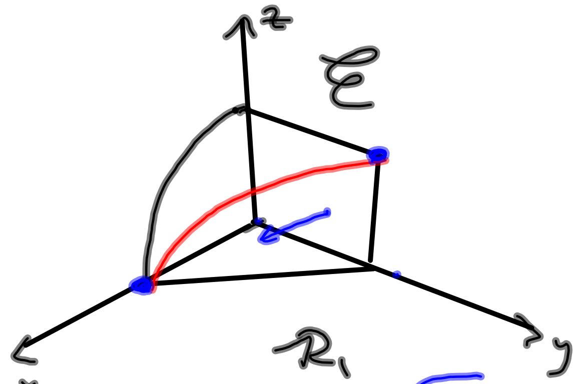
$$y-1 = \pm\sqrt{1-z}$$

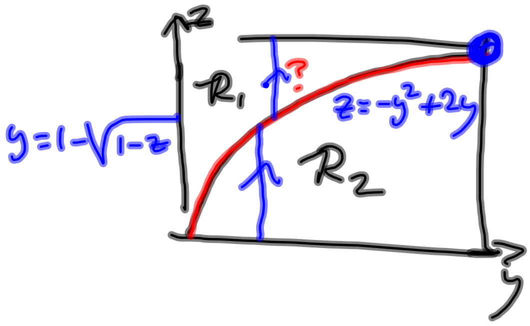
$$y = 1\pm\sqrt{1-z}$$

which one? $\pm$?

<u>The negative</u>

Apr 4-12:35 PM





$\mathcal{R}_1$

$$\int_0^1 \int_{z=-y^2+2y}^{z=1} \int_{x=0}^{x=\sqrt{1-z}} f \, dx \, dz \, dy$$

$\mathcal{R}_2$

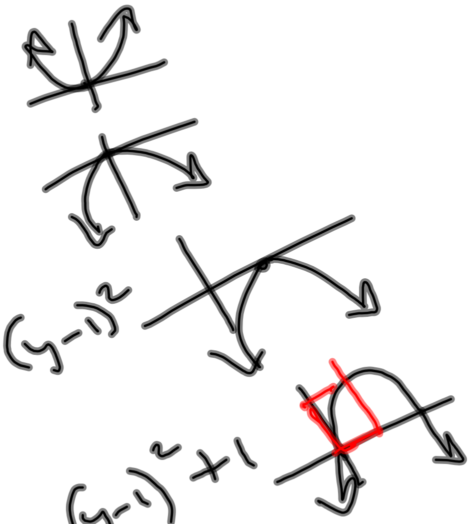
$$+ \int_0^1 \int_0^{-y^2+2y} \int_0^{1-y} f \, dx \, dz \, dy$$

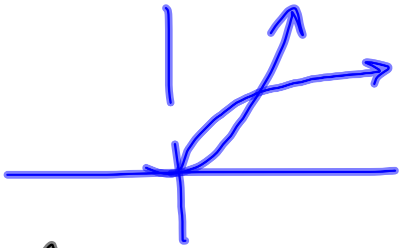
$z = y^2$

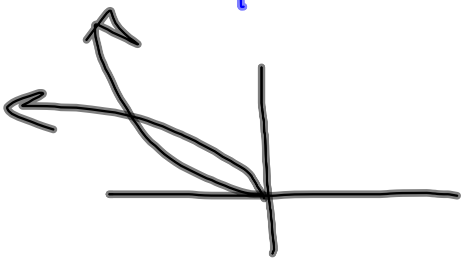
$-y^2$

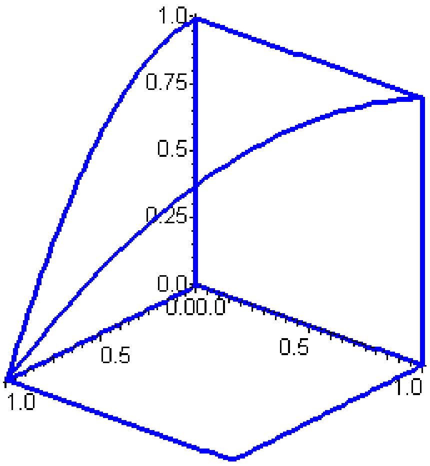
$-(y-1)^2$

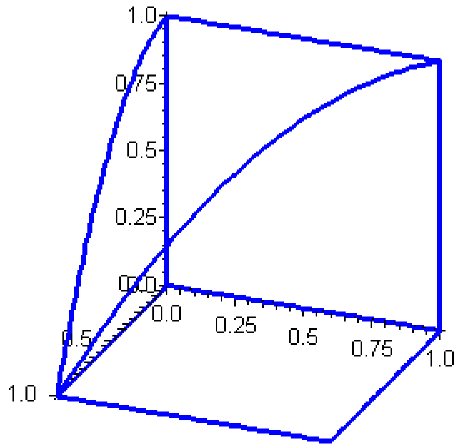
$-(y-1)^2+1$

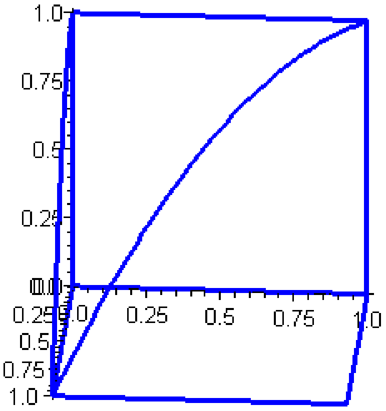


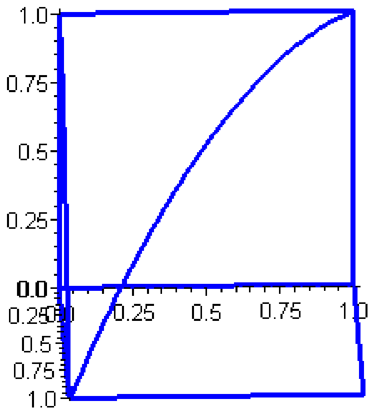


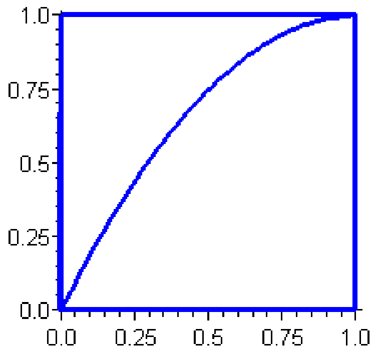


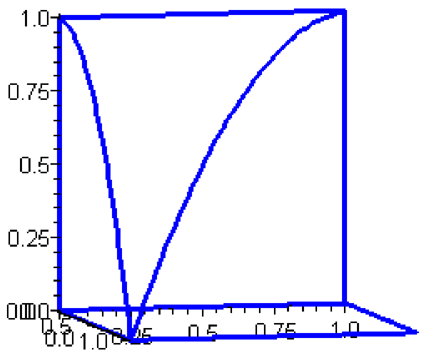

1.0
0.75
0.5
0.25
0.0
0.0
0.0
0.5
0.5
1.0
1.0


1.0
0.75
0.5
0.25
0.0
0.0
0.25
0.5
0.75
1.0
0.5
1.0


1.0
0.75
0.5
0.25
0.0
0.25
0.5
0.75
1.0
0.0
0.25
0.5
0.75
1.0


1.0
0.75
0.5
0.25
0.0
0.25
0.5
0.75
1.0
0.25
0.5
0.75
1.0


1.0
0.75
0.5
0.25
0.0
0.0
0.25
0.5
0.75
1.0


1.0
0.75
0.5
0.25
0.5
0.75
1.0