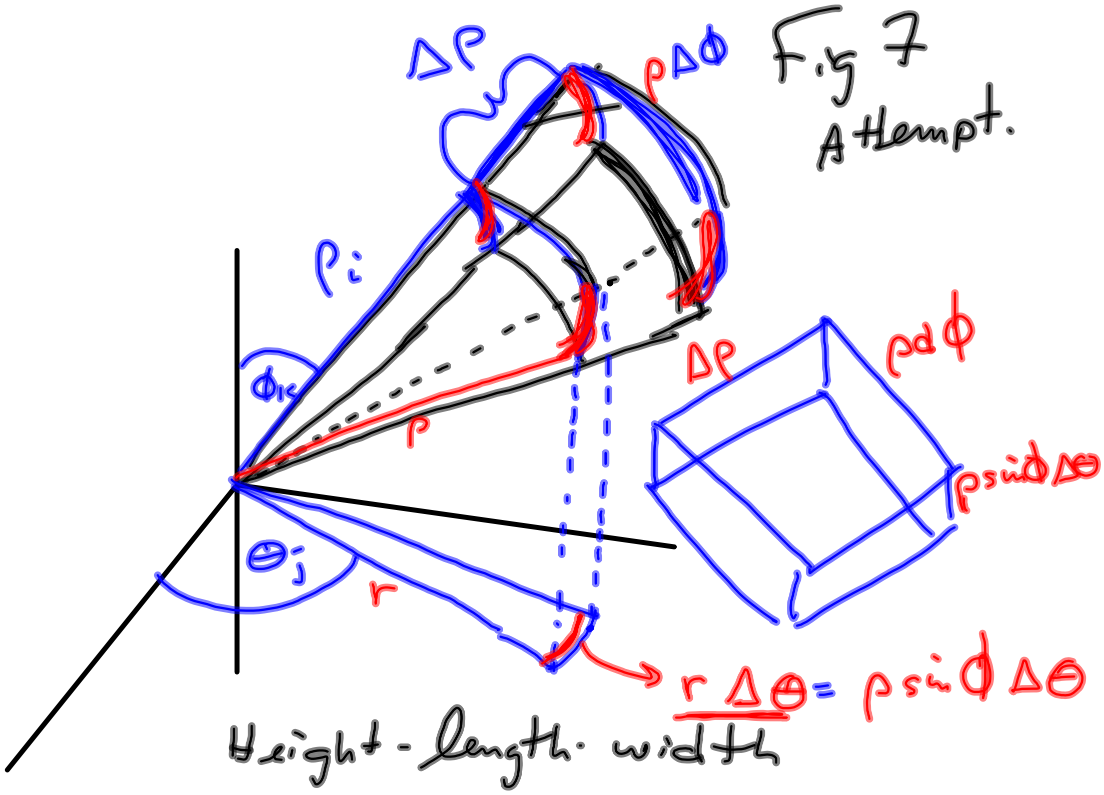
Fig 7
Attempt.



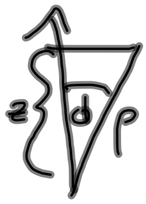
Height · length · width

$= \rho d\phi\, \rho \sin\phi\, \Delta\theta\, \Delta\rho$

$= \rho^2 \sin\phi\, \Delta\rho\, \Delta\theta\, \Delta\phi$

$=$ Volume

So $\iiint\limits_{E} f(x,y,z)\,dV$ is



$$\int_c^d\int_\alpha^\beta\int_a^b \underbrace{f(\rho\sin\phi\cos\theta,\ \rho\sin\phi\sin\theta,\ \rho\cos\phi)}\ \underline{\rho^2\sin\phi\, d\rho\, d\theta\, d\phi}$$

$$E = \{(\rho,\theta,\phi) \mid a\le\rho\le b,\ \alpha\le\theta\le\beta,\ c\le\phi\le d\}$$

SEE TEC for this.

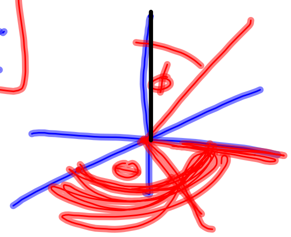
§16.6 #26 is Bonus

~~16.8 #11~~

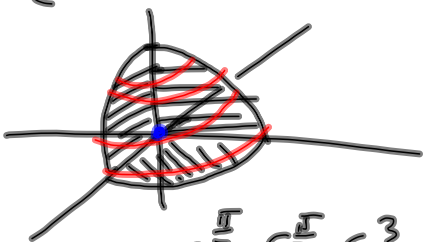
16.8 #24

$$\iiint_E e^{\sqrt{x^2+y^2+z^2}}\,dV,$$ where

E: $\boxed{x^2+y^2+z^2=9}$ region enclosed by this sphere of radius 3, centered @ origin, $\boxed{\text{in } 1^{st} \text{ octant.}}$ → $\boxed{\begin{array}{l}\phi \le \frac{\pi}{2}\\ \theta \le \frac{\pi}{2}\end{array}}$



$$\iiint_E e^{\rho}\,\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$



$0 \le \phi \le \frac{\pi}{2}$

$0 \le \phi \le \frac{\pi}{2}$

$0 \le \rho \le 3$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^{3} e^{\rho}\rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$$

$$= \frac{\pi}{2}\left(5e^3-2\right)$$

$$\int_0^3 e^{\rho}\rho^2\,d\rho = \left[\rho^2e^{\rho} - 2\rho e^{\rho} + 2e^{\rho}\right]_0^3 = 9e^3-6e^3+2e^3-2 = 5e^3-2$$

$u=\rho^2 \quad du = 2\rho\, d\rho$

$dv = e^{\rho}d\rho \quad v = e^{\rho}$

$$\int \rho^2 e^{\rho}\,d\rho$$

$$= uv - \int v\,du = \rho^2e^{\rho} - \boxed{\int 2\rho e^{\rho}\,d\rho}$$

$$\boxed{2\int \rho e^{\rho}\,d\rho}$$

$u=\rho \quad du=d\rho$

$dv = e^{\rho}d\rho \Rightarrow v = e^{\rho}$

$$= 2\rho e^{\rho} - 2\int e^{\rho}\,d\rho = \boxed{2\left[\rho e^{\rho}-e^{\rho}\right]}$$

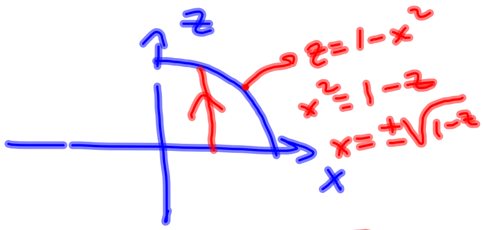
16.6 #34

$$\int_0^1 \int_0^{1-x^2} \left[\int_0^{1-x} f(x,y,z)\, dy \right] dz\, dx$$

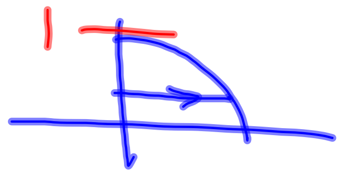
Outer two integrals are integrating

$$G(x,z) = \int_0^{1-x} f(x,y,z)\, dy$$

$$\int_0^1 \int_0^{1-x^2} G(x,z)\, dz\, dx$$



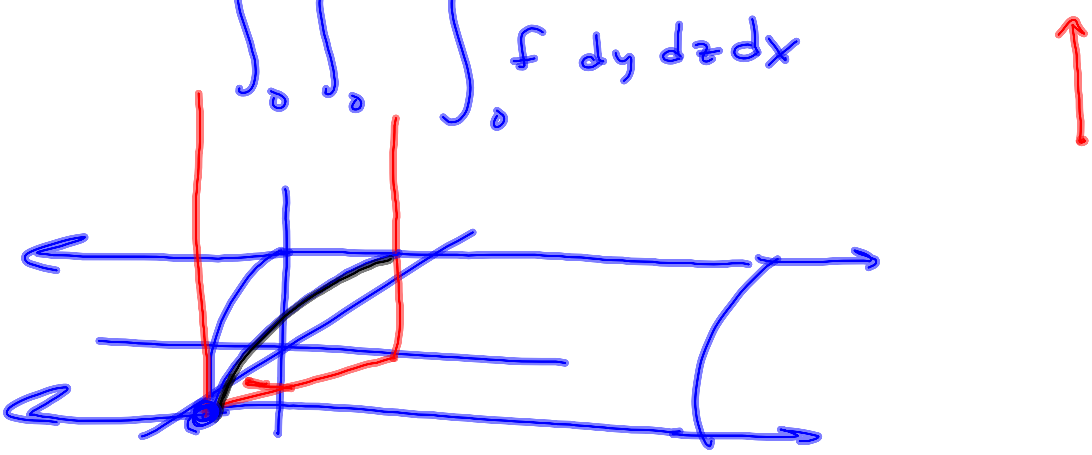
How to swap these:



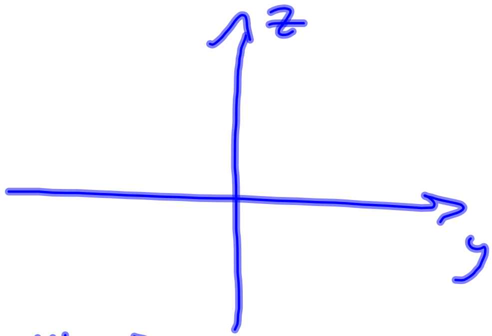
$$= \int_0^1 \int_0^{\sqrt{1-z}} G(x,z)\, dx\, dz$$

$$= \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x,y,z)\, dy\, dx\, dz$$

$$\int_0^1 \int_0^{1-x^2} \int_0^{1-x} f\, dy\, dz\, dx$$



$$\int_0^1 \left[\int_{0=z}^{1-x^2=z} \int_{0=y}^{1-x=y} f \, dy \, dz \right] dx$$



§16.8 #s 2, 4, 6, 8, 12, 14, 15, 18, 26, 36

§16.9

$z = 1 - x^2$

$x = \sqrt{1-z}$

$y = 0, \ldots, 1 - \left((1-z)^{\frac{1}{2}}\right)^2 = 1 - x^2$

$= 1 - (1-z)$

$= z$