16.4 #32 TI

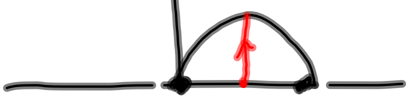
$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

To see "it" $D = \{(x,y) \mid 0 \le x \le 2,\ 0 \le y \le \sqrt{2x-x^2}\}$

Draw this

$\sqrt{2x-x^2} = \sqrt{x(2-x)}$

what's it, really?



$y = \sqrt{2x-x^2}$

$y^2 = 2x - x^2$

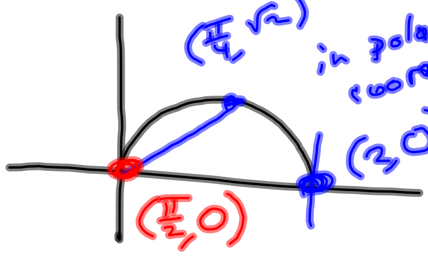
$x^2 - 2x + y^2 = 0$

$x^2 - 2x + 1 + y^2 = 1$

$(x-1)^2 + y^2 = 1$

So this is the top half of this circle:

Polar coordinates.



$y^2 = 2x - x^2$

$x^2 + y^2 = 2x$

$r^2 = 2r\cos\theta$

$r = 2\cos\theta$

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

$$= \int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r \cdot r\, dr\, d\theta$$

#36a

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\,dA = \int_{x=-\infty}^{x=\infty}\int_{y=-\infty}^{y=\infty} e^{-x^2-y^2}\,dy\,dx$$

$$\int_0^{2\pi}\int_0^{\infty} e^{-r^2}\,r\,dr\,d\theta$$

$u = -r^2$

$du = -2r\,dr$

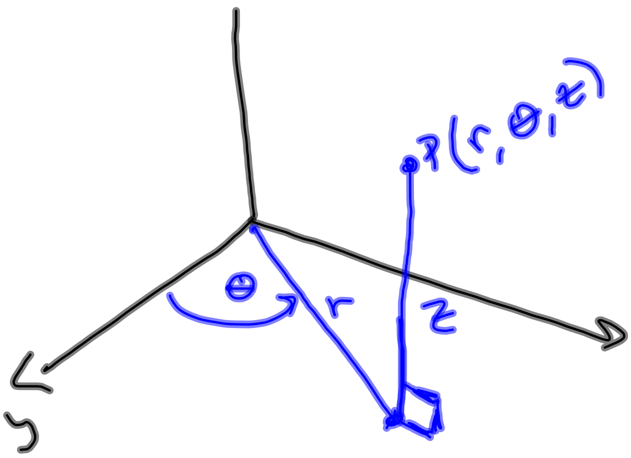
$$= \lim_{a\to\infty}\int_0^{2\pi} -\frac{1}{2}\int_0^{a} e^{-r^2}\left(-2r\,dr\,d\theta\right)$$

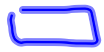
$$= \lim_{a\to\infty}\left(-\frac{1}{2}\int_0^{2\pi}\left[e^{-r^2}\right]_0^{a} d\theta\right)$$

Nice problem to see for complex analysis to come. (Improper Integrals)

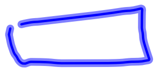
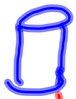
$= \text{etc.}$

§16.7 Cylindrical Coordinates.



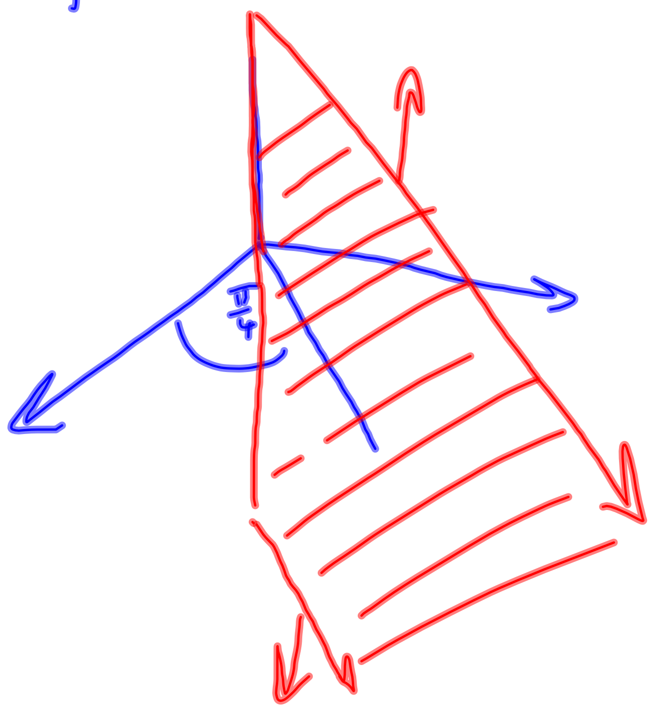
 to 

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$

 to 

$r^2 = x^2 + y^2,\ \tan\theta = \frac{y}{x},\ z = z$

In cylindrical coordinates, $\theta = \frac{\pi}{4}$ is a half-plane.



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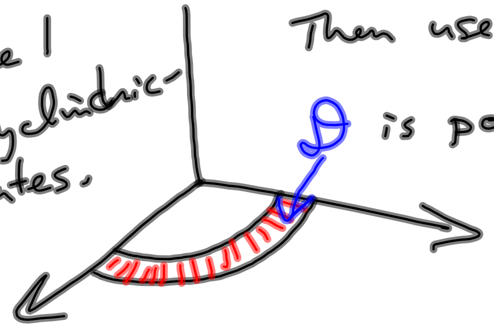
Let $\mathcal{E}$ be a solid region

If $\mathcal{E}$'s projection into the xy-plane

is handled nicely nicely by polar coordinates

Then use 'em:

The Type I take on cylindrical coordinates.



$\mathcal{D}$ is polar-coordinate friendly.

$$\mathcal{E} = \{ (x,y,z) \mid (x,y) \in \mathcal{D},\ u_1(x,y) \le z \le u_2(x,y) \}$$

§16.6 says

$$\iiint_{\mathcal{E}} F(x,y,z)\,dV = \iint_{\mathcal{D}} \underbrace{\left[\int_{z=u_1(x,y)}^{z=u_2(x,y)} F(x,y,z)\,dz \right]}_{\text{is the } f(x,y) \text{ from } 16.6.} dA$$

If $\mathcal{D} = \{ (r,\theta) \mid \alpha \le \theta \le \beta,\ h_1(\theta) \le r \le h_2(\theta) \}$, then we have

$$\int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} f(r,\theta) \quad r\,dr\,d\theta$$

is a 16.4 question, now.

$$f(r,\theta) = \int_{z=u_1(r\cos\theta,\, r\sin\theta)}^{z=u_2(r\cos\theta,\, r\sin\theta)} F(r\cos\theta, r\sin\theta, z)\,dz$$

FOR NEXT TIME, E4 from §16.7

§16.6 due Tuesday.

§ " 7 .. Wednesday

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§16.6 #3

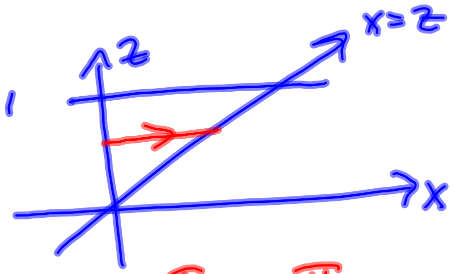
$$\int_0^1 \int_0^z \int_0^{x+z} 6xz \, dy \, dx \, dz = 1$$

The projection of that solid into the xz-plane.

Indicates Type 3 solid

This integral is $f(x,z)$ living over the xz-plane.

$$\int_{z=0}^{z=1} \int_{x=0}^{z}$$



Type II

Drawing these pictures is key to switching order of integration when the limits of integration are variable.