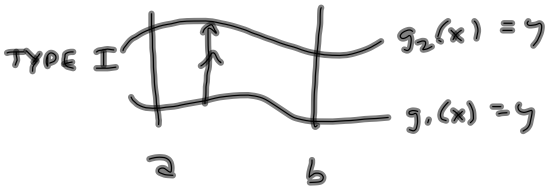
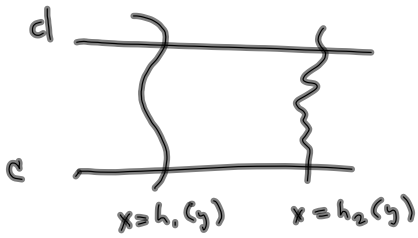
16.3 Pics "I"



$$\int_{x=a}^{x=b} \int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\,dy\,dx$$

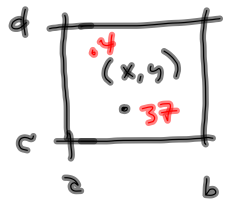


$$\int_{y=c}^{y=d} \int_{x=h_1(y)}^{x=h_2(y)} f(x,y)\,dx\,dy$$

§16.6 Triple Integrals over a Rectangular Box

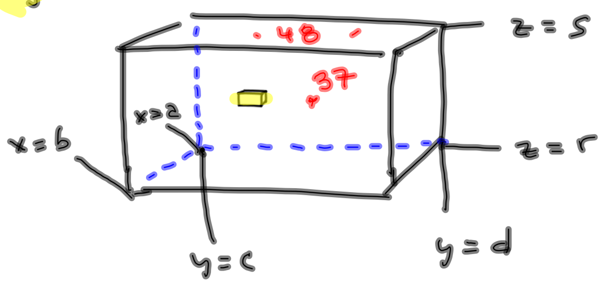
$$\mathcal{B} = \{(x,y,z) \mid a \le x \le b,\ c \le y \le d,\ r \le z \le s\}$$

$$= [a,b] \times [c,d] \times [r,s]$$



$(x,y) \in [a,b] \times [c,d]$

$$\mathcal{B}_{ijk} = [x_{i-1}, x_i] \times [y_{j-1}, y_j] \times [z_{k-1}, z_k]$$



$f(x_i, y_j, z_k)$ corresponds

$f(x)$

$(x, f(x))$

$f(x,y)$

$(x, y, f(x,y))$

$\boxed{(x, y, z,}\, f(x,y,z))$

Triple Riemann Sum

$$\sum_{i=1}^{l} \sum_{j=1}^{m} \sum_{k=1}^{n} f(x_{ijk}^*, y_{ijk}^*, z_{ijk}^*) \Delta V$$

If boxes are "small enough"

$$\sum\sum\sum f(x_i, y_j, z_k)$$

$$\xrightarrow{l,m,n \to \infty} \iiint_B f(x,y,z)\, dV$$

Continuity of f assures existence of $\iiint$ as a number.

Fubini's for integrals

$$\int_i \int_j \int_k = \int_i \int_k \int_j = \int_j \int_i \int_k$$
$$= \int_j \int_k \int_i = \int_k \int_i \int_j = \int_k \int_j \int_i$$

Order of integration doesn't matter.

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Type 1 : $\mathcal{E}$ = Region (Not just rectangular box.)

See Fig 2

$z = u_1(x,y)$ below $\mathcal{E}$

$z = u_2(x,y)$ above $\mathcal{E}$

$\mathcal{D}$ = the projection of $\mathcal{E}$ onto the xy-plane.

$$\iiint_{\mathcal{E}} f\,dV = \iint_{\mathcal{D}} \left[\int_{z=u_1(x,y)}^{z=u_2(x,y)} f(x,y,z)\,dz \right] dA = \iint_{\mathcal{D}} G(x,y)\,dA$$

$$\boxed{\int_{z=u_1(x,y)}^{z=u_2(x,y)} f(x,y,z)\,dz}$$ is in the role of $F(x,y)$ in 16.3 !

IS a function of x & y.

$$\iint_{\mathcal{D}} G(x,y)\,dA$$

$\mathcal{D}$ is the projection of $\mathcal{E}$ onto the xy-plane & it is of Type I or Type II

Type 1, I

Type 1, II

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Type 2: $x = u_1(y,z)$ left bound
$x = u_2(y,z)$ right bound

$\mathcal{D}$ is projection of $\mathcal{E}$ into the yz-plane.

$$\boxed{\iint_{\mathcal{D}}} \underbrace{\left[\int_{x=u_1(y,z)}^{x=u_2(y,z)} f(x,y,z)\, dx \right]}_{\text{Like } f(x,y) \text{ in } \S 16.3} \boxed{dA}$$

Type 3

$y = u_1(x,z)$ back
$y = u_2(x,z)$ front

Type 3, I

Type 3, II

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#12 view E as type 1

#13 ① There's a $\iint f(x)g(y)\,dy\,dx$

$= \int f(x)\,dx \int g(y)\,dy$ step.

② Integrate by parts, twice.

#33 Good praxis

#17 Polar coords

§16.7