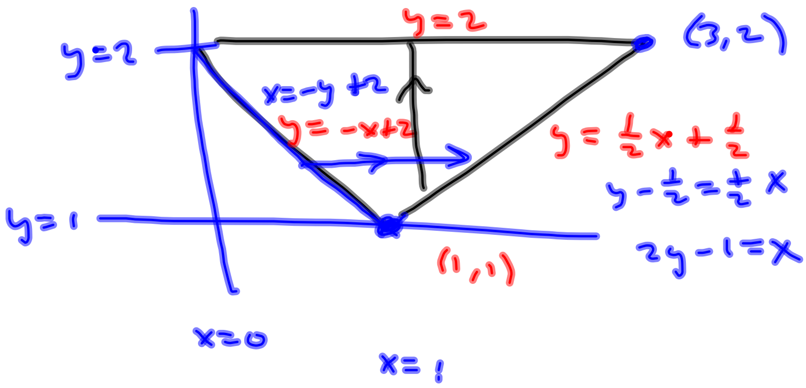
16.3 #15



Type I

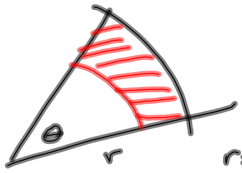
$$\int_0^1\int_{-x+2}^{2} \sim \, dy\, dx + \int_1^3\int_{\frac{1}{2}x+\frac{1}{2}}^{2} \sim \, dy\, dx$$

Type II

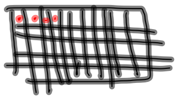
$$\int_1^2\int_{-y+2}^{2y-1} \sim \, dx\, dy$$

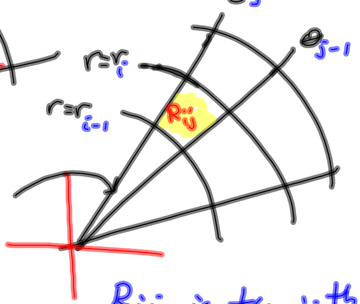
§16.4

Area of sector



$A = \frac{1}{2} r^2 \theta$





Its area is

$\frac{1}{2} r_i^2 (\theta_j - \theta_{j-1}) - \frac{1}{2} r_{i-1}^2 (\theta_j - \theta_{j-1})$

R_{ij} is the ij^{th} polar rectangle.

$= \frac{1}{2} (r_i^2 - r_{i-1}^2)(\theta_j - \theta_{j-1})$

$= \frac{1}{2} (r_i^2 - r_{i-1}^2) \Delta\theta$

$\sum f(x) \Delta x$

$\boxed{\sum f(x_i) \Delta x}$

$= \boxed{\frac{1}{2}(r_i + r_{i-1})} (r_i - r_{i-1}) \Delta\theta$

$= \quad r_i^* \quad (r_i - r_{i-1}) \Delta\theta$

$\bar{r}_i$ = average of the two: $\frac{r_i + r_{i-1}}{2}$

$= r_i \Delta r \theta \Delta$

Volume of a function $f(x,y)$ over the ij^{th} "rectangle"

$\sum_{j=1}^{n} \sum_{i=1}^{m} f(r_i, \theta_j)\, r_i \Delta r \Delta\theta$

If you wanted the whole deal.

Recall

$x = r\cos\theta, \; y = r\sin\theta, \; x^2 + y^2 = r^2$

So volume under the function over a domain $\{(r,\theta) \mid r_1 \le r \le r_2, \; \theta_1 \le \theta \le \theta_2\}$

$$\int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} f(r,\theta)\, r \, dr \, d\theta$$

BTW

$$f(x,y) = g(x)\,h(y)$$

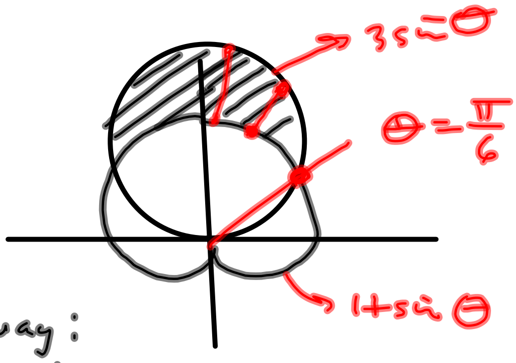
$$\iint (x^2+2)(y^2-7)\,dx\,dy$$

Theorem undiscussed. Pg 999

$$= \int (x^2+2)\,dx \int (y^2-7)\,dy$$

$$\text{Area} = \iint 1\,dx\,dy$$

Find area inside $r = 3\sin\theta$
and outside $r = 1+\sin\theta$



Area of sector:
$\frac{1}{2}r^2\theta$

11.4 way:

$$2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\frac{1}{2}\left[3\sin\theta\right]^2 - \frac{1}{2}\left[1+\sin\theta\right]^2\right)d\theta$$

16.4 way

$$\text{Area} = \iint f(r,\theta)\, r\, dr\, d\theta$$

$$= 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_{1+\sin\theta}^{3\sin\theta} 1\, r\, dr\, d\theta$$

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$$\int_{\frac{Pi}{6}}^{\frac{Pi}{2}} \left((3\cdot\sin(x))^2 - (1+\sin(x))^2 \right) dx$$

Proof by Maple!

Study the derivation some.

See Fig. 4

π

$$2\cdot\int_{\frac{Pi}{6}}^{\frac{Pi}{2}} \left(\int_{1+\sin(t)}^{3\cdot\sin(t)} r\,dr \right) dt$$

3-leaved clover

π

3

5

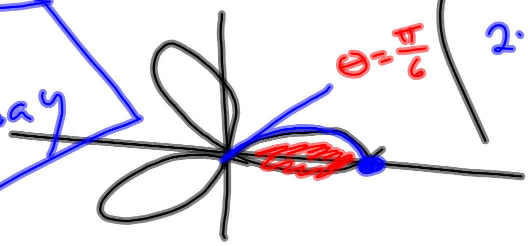
$\pi r^2 = \frac{2\pi}{2} r^2$

$r = \cos(3\theta)$

Find area of one loop.

$\S$16.4 Due Friday

$\theta = \frac{\pi}{6}$



$$2\cdot\frac{1}{2}\cdot\int_0^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (1+\cos(6\theta))\,d\theta = \frac{1}{2}\left[\theta + \frac{1}{6}\sin(6\theta)\right]_0^{\frac{\pi}{6}}$$

$$= \frac{1}{2}\left[\frac{\pi}{6} + \underline{\frac{1}{6}\cdot 0} - (0-0)\right] = \frac{\pi}{12}$$

16.4 way

$$2\int_0^{\frac{\pi}{6}}\int_0^{\cos(3\theta)} 1\, r\,dr\,d\theta = 2\int_0^{\frac{\pi}{6}} \left[\frac{1}{2}r^2\right]_0^{\cos(3\theta)} d\theta$$

$$= \int_0^{\frac{\pi}{6}} \cos^2(3\theta)\,d\theta = \text{Same thing.}$$