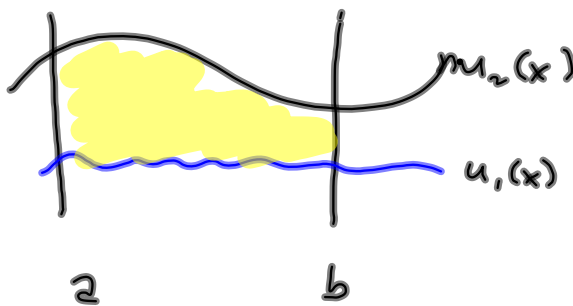


## §1.6.2 Iterated Integrals

$$\begin{aligned} & \int_0^1 \int_1^2 (4x^3 - 9x^2y^2) dy dx \\ &= \int_0^1 \left[ 4x^3y - 3x^2y^3 \right]_{y=1}^{y=2} dx \\ &= \int_0^1 (8x^3 - 24x^2 - [4x^3 - 3x^2]) dx \\ &= \int_0^1 (4x^3 - 21x^2) dx = \left[ x^4 - 7x^3 \right]_0^1 = 1 - 7 = -6 \end{aligned}$$

### §16.3 Integrating over more general regions

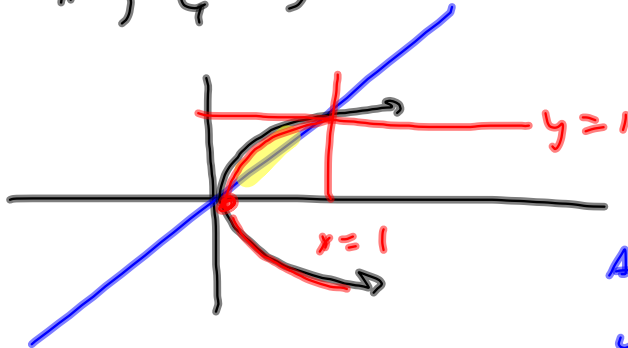
Type I



$z = f(x, y)$  is a surface defined for all  $(x, y) \in \mathcal{D}$ .

$$\iint_{\mathcal{D}} f(x, y) \, dA = \int_a^b \int_{u_1(x)}^{u_2(x)} f(x, y) \, dy \, dx$$

Find volume of region bounded  
by  $z = 2x + y^2$ , above the region bdd. by  
 $x = y$  &  $x = y^2$



$$y = y^2 \Rightarrow y = \pm 1$$

As a type I region:

$y = x$  is lower =  $u_1(x)$

$y = \sqrt{x}$  is upper =  $u_2(x)$

Not nice  
For type I approach want  $+\sqrt{x}$

$$x = y^2 = x \quad y = \pm \sqrt{x} \quad \int_0^1 \int_{y=x}^{y=\sqrt{x}} (2x+y^2) dy dx$$

$$= \int_0^1 \left[ 2xy + \frac{1}{3}y^3 \right]_{y=x}^{y=\sqrt{x}} dx$$

$$= \int_0^1 \left[ 2x\sqrt{x} + \frac{1}{3}x^{\frac{3}{2}} - \left( 2x^2 + \frac{1}{3}x^3 \right) \right] dx$$

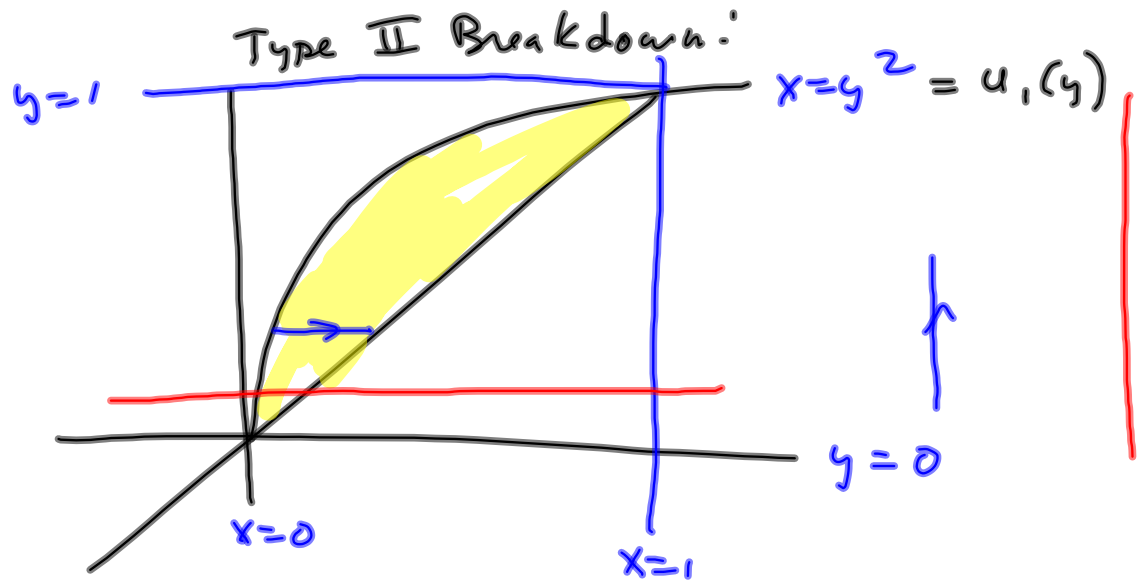
$$= \int_0^1 \left[ \frac{7}{3}x^{\frac{3}{2}} - 2x^2 - \frac{1}{3}x^3 \right] dx$$

$$= \left[ \frac{7}{3} \cdot \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^3 - \frac{1}{12}x^4 \right]_0^1$$

$$= \frac{14}{15} - \frac{2}{3} - \frac{1}{12} = \frac{56 - 40 - 5}{60} = \boxed{\frac{11}{60}}$$

Type I kinda sucked,

Type II holds promise!



$$\int_0^1 \int_{x=y^2}^{x=y} (2x + y^2) dx dy$$

$$= \int_0^1 [x^2 + xy^2]_{x=y^2}^{x=y} dy$$

$$= \int_0^1 [\underline{y^2 + y^3} - (\underline{y^4 + y^4})] dy$$

$$= \int_0^1 [y^2 + y^3 - 2y^4] dy$$

$$= \left[ \frac{1}{3} y^3 + \frac{1}{4} y^4 - \frac{2}{5} y^5 \right]_0^1$$

$$= \frac{1}{3} + \frac{1}{4} - \frac{2}{5}$$

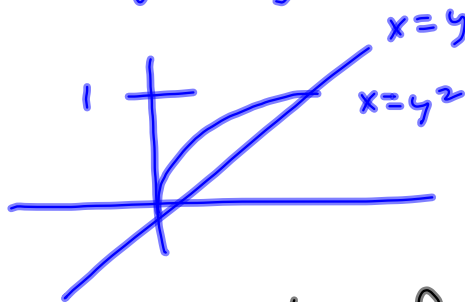
$$= \frac{20 + 15 - 24}{60} = \frac{11}{60}$$

$$\int_0^1 \int_{x=y^2}^{x=y} (2x+y^2) dx dy = \int_0^1 \int_{y=x}^{y=\sqrt{x}} (2x+y^2) dy dx$$

Later in the chapter, we'll be switching the order of integration

$$\int_0^1 \int_{x=y^2}^{x=y} f(x,y) dx dy$$

Integrating over a Type II region.

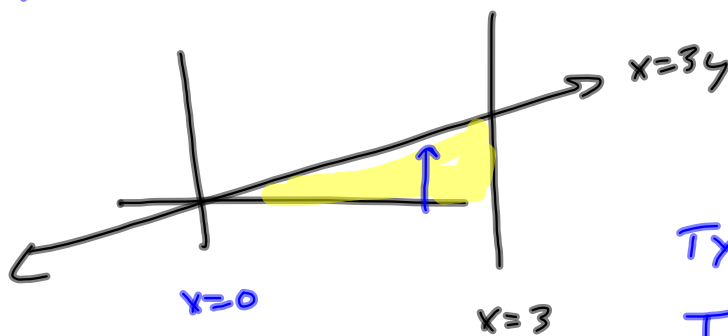


To swap order of integration, we need to convert to a Type I.

Example 3 is cool, because the Type I requires 2 integrals, because the lower function  $g_1(x)$  changes @  $x = -1$ .

(45) Viewed as Type II  $\int_0^1 \int_{x=3y}^{x=3} e^{x^2} dx dy$

$e^{x^2}$  has no closed-form antiderivative.



Type I approach:

This gives  $\int_0^3 \int_{y=0}^{\frac{1}{3}x} e^{x^2} dy dx$

Type I:  $g_1(x) = 0$   
 $g_2(x) = \frac{1}{3}x$

$$= \int_0^3 \left[ y e^{x^2} \right]_{y=0}^{y=\frac{1}{3}x} dx$$

$$= \int_0^3 \left[ \frac{1}{3} x e^{x^2} \right] dx = \frac{1}{3} \cdot \frac{1}{2} \int_0^3 e^{x^2} \cdot 2x dx$$

$$= \left[ \frac{1}{6} e^{x^2} \right]_0^3 = \boxed{\frac{1}{6} e^9 - \frac{1}{6}} \text{ Sweet!}$$

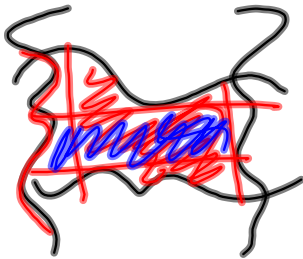
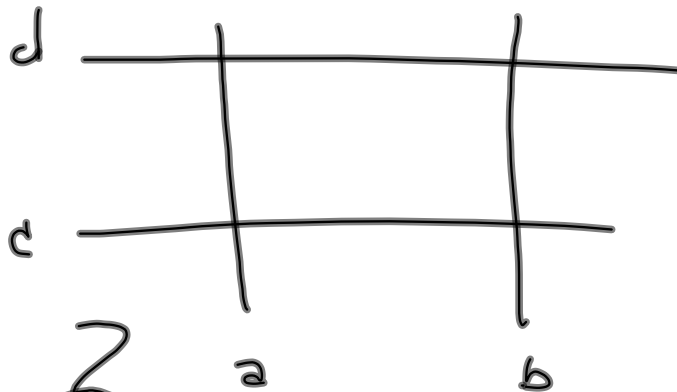
Fubini's Theorem

$$\int_a^b \int_c^d f(x,y) dy dx$$

$$= \int_c^d \int_a^b f(x,y) dx dy$$

Only works over rectangles:

$$[a,b] \times [c,d]$$



$$\frac{d}{dx} \int_a^x \int_b^y g(t,s) ds dt$$

$$= \int_b^y g(x,s) ds$$

$$\frac{d}{dx} \int_a^x f(s) ds = f(x)$$