16.1 #s 1, 5, 7, 8, 11, 14

16.2 #s 1, 4, 7, 10, 15, 19, 27, 31, 38

16.2 #s 1, 8, 15, 19, 20, 31, 48, 53

16.4 #s 2, 4, 5, 8, 11, 15, 18, 32, 362

16.5 #s

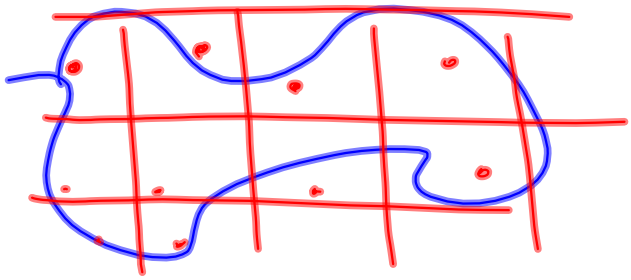
16.6 #s 10, 12, 13, 15, 19, 26 B

16.7 #s 1–13, 15, 18, 20, 27

16.1 Volume over a rectangle

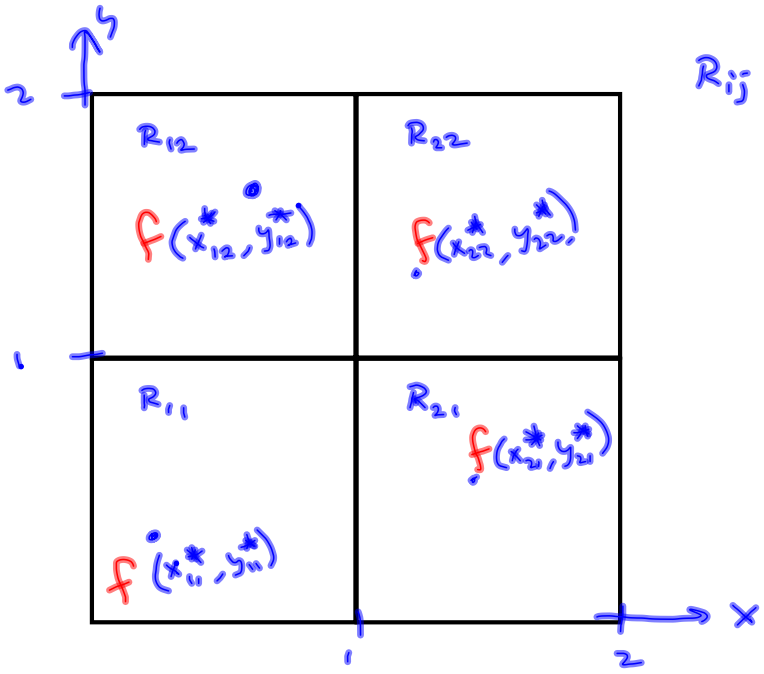
$z = f(x,y)$ assigns a number to each point (x,y) in some rectangle, R.
It's a surface over a rectangle.

Underground oil reservoir.



Each drill hole gives you a sample depth for reservoir.

Depth · Area gives amount of oil in each square in the grid.
Add up these volumes.



R_{ij} = ij^{th} rectangle

$$\text{Volume} \approx \left(f(x_{11}^*, y_{11}^*) + f(x_{21}^*, y_{21}^*) + \cdots + f(x_{22}^*, y_{22}^*)\right)\Delta A$$

$$= \sum_{i=1}^{2}\left(\sum_{j=1}^{2} f(x_{ij}^*, y_{ij}^*)\right)\Delta A$$

Iterated sum

Hold i constant, run thru the j's. Then sum thru the i's.

$$= \left(f(x_{11}^*, y_{11}^*) + f(x_{12}^*, y_{12}^*) + f(x_{21}^*, y_{21}^*) + f(x_{22}^*, y_{22}^*)\right)\Delta A$$

Goal: $m, n \longrightarrow \infty$ $\longrightarrow$ $\int_0^1\int_0^1 f(x,y)\,dy\,dx$

We'll be doing this by taking (x_{ij}^*, y_{ij}^*) to be

1. upper right corner
2. midpoint.

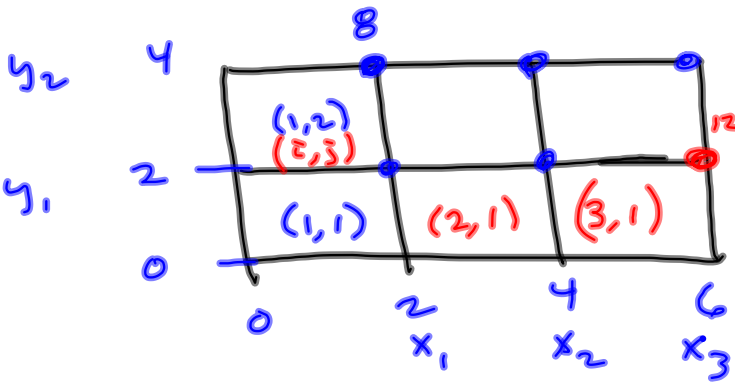
$f(x_i, y_j)$

$$R = \{(x,y) \mid 0 \le x \le 6,\ 0 \le y \le 4\}$$

$$\Delta y = \Delta x = 2$$

$f(x,y) = xy$

$z = xy$



$$\sum_{i=1}^{3}\sum_{j=1}^{2} f(x_i^*, y_j^*)\,\Delta y\,\Delta x$$

$$= \Big[f(x_1,y_1) + f(x_1,y_2) + f(x_2,y_1) + f(x_2,y_2) + f(x_3,y_1) + f(x_3,y_2) \Big] (4)$$

$$(m,n) \to (\infty,\infty) \longrightarrow \int_{x=0}^{x=6}\int_{y=0}^{y=4} xy\,dy\,dx$$

16.2

Recall Clairaut's Theorem:

$$f_{xy} = f_{yx}$$

Fubini's Theorem

$$\int_a^b \int_c^d f(x,y)\,dy\,dx$$

$$= \int_c^d \int_a^b f(x,y)\,dx\,dy$$

$$\sum_{i=1}^{3}\sum_{j=1}^{2} x_i y_j = x_1y_1 + x_1y_2 + x_2y_1 + x_2y_2 + x_3y_1 + x_3y_2$$

$$= x_1y_1 + x_2y_1 + x_3y_1 + x_1y_2 + x_2y_2 + x_3y_2$$

$$= \sum_{j=1}^{2}\sum_{i=1}^{3} x_i y_j$$

Add #22 to assignment

§16.1 Monday

§16.2 Tuesday One ugly integral that exercises your Calc II muscles.

#22 is tough!

$$\int \frac{2y^2}{y^2+4}\,dy$$

$$\frac{1}{2}\ln\left(\frac{5}{2}\right) + 2\arctan\left(\frac{1}{2}\right) - \frac{\pi}{2}$$