§ 15.8 # 41

$f(x,y,z) = x^2+y^2+z^2$ Bleah.

s.t.

$x+y+2z=2$

$g(x,y,z) = x+y+2z-2$

$\nabla f = \lambda \nabla g$ Something off.

$2x = \lambda$ $2y = \lambda$ $2z = 2\lambda$

$x = \frac{\lambda}{2}$ $y = \frac{\lambda}{2}$ $z = \lambda$ ?

$\frac{\lambda}{2} + \frac{\lambda}{2} + 2\lambda = 2$

$3\lambda = 2$

$\lambda = 2/3$

$x = \frac{1}{3}, y = \frac{1}{3}, z = \frac{2}{3}$

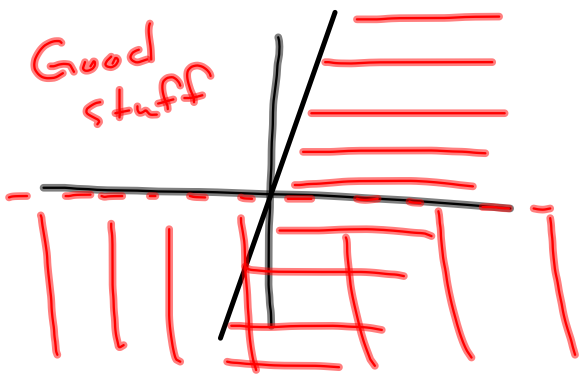
15.1 $\mathcal{D}$ :

$$F(x,y) = \sqrt{2x-y}\,\ln(y)$$

Need $2x - y \geq 0$ Need $y > 0$

$$-y \geq -2x$$

$$y \leq 2x$$



15.2 Some sort of limit.

15.3 Find $1^{st}$ partials $f_x$, $f_y$

$f(x,y) = \cos(3x-2y)$

$f_x = -\sin(3x-2y)(3)$

$f_y = -\sin(3x-2y)(-2)$

$f(x,y) = -\int_{x^3}^{y} \operatorname{arcsec}\sqrt[3]{\tan(t^2-t^{\frac{1}{2}})}\,dt$

$f_x = \left(\operatorname{arcsec}\sqrt[3]{\tan(x^2-x^{\frac{1}{2}})}\right)(3x^2)$

$f_y = -\operatorname{arcsec}\sqrt[3]{\tan(y^2-y^{\frac{1}{2}})}$

$\int$ 15.3 #28 is bogus.

$-f(x))$

$f(x,y) = \int_y^x \cos(t^2)\,dt \Rightarrow$

$f_x = \cos(x^2) = \lim_{h\to 0}\left[\dfrac{\int_y^{x+h}\cos(t^2)\,dt - \int_y^x \cos(t^2)\,dt}{h}\right]$

$f_y = -\cos(y^2)$

15.3 & 15.5

Find $\frac{\partial z}{\partial x}$ for

$x^2+y^2-5z^2 = 11xy$

(i) Implicit Diff 15.3

(ii) By 15.5 technique $\quad -\dfrac{F_x}{F_z} = \dfrac{\partial z}{\partial x}$

Mar 19-12:16 PM

15.4 & 15.6 go together.

$z = 2x^2 - y^2$. Find eq'n of tangent plane to this surface by @ $(1,2,0)$

15.4 technique

$$z = f_x(x - x_0) + f_y(y - y_0) + z_0$$

$f_x = 4x$ $\quad f_x(1,2) = 4$

$f_y = -2y$ $\quad f_y(1,2) = -4$

$$z = 4(x-1) - 4(y-2) + 0$$

15.6 Technique.

$z = 2x^2 - y^2 = f(x,y)$ is NOW viewed as a level ~~curve~~ surface for

$$F(x,y,z) = 2x^2 - y^2 - z$$

and the plane has eqn

$$F_x(x - x_0) + F_y(y - y_0) + F_z(z - z_0) = 0$$

$F_x = 4x$ $\quad F_x(1,2) = 4$

$F_y = -2y$ $\quad F_y(1,2) = -4$

$F_z = -1$

So $\quad 4(x-1) - 4(y-2) - 1(z-0) = 0$

$\Longrightarrow \quad z = 4(x-1) - 4(y-2)$

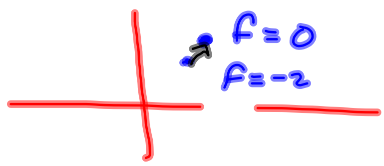
Use it to approximate

$f(1.1, 2.1)$ $\quad f(x,y) = 2x^2 - y^2$

$\approx L(1.1, 2.1) = 4(1.1-1) - 4(2.1-2)$

$= 4(.1) - 4(.1) = 0$

Find $\Delta f$




$\Delta f = f(1.1, 2.1) - f(1,2)$

$= 0 - (-2) = 2 = \Delta f.$

Use $df$ to approximate $\Delta f$

$df = f_x\, dx + f_y\, dy$

$= 4x\, dx - 2y\, dy$

$x = 1,\ y = 2,\ dx = dy = .1$

$\Rightarrow df = 4(1)(.1) - 2(2)(.1)$

$= .4 - .4 = 0$

Ugh. Not good
The test question
I wrote is
MUCH nicer.

Test 2 parts

Part 1 15.1 – 15.6

15.6's tangent plane thingie in Part 1.

Part 2 15.6 – 15.8

15.6's directional derivative thing.

Some basic vector maneuvering / navigation.

Move 5 units in the $\langle 3,4,0\rangle$ direction from $\langle 2,-7,11\rangle$

$$\frac{\langle 3,4,0\rangle}{\sqrt{25}} = \frac{1}{5}\langle 3,4,0\rangle$$

Unit vector in direction of $\langle 3,4,0\rangle$ is $\frac{1}{5}\langle 3,4,0\rangle$

Now go 5 units in that direction from $\langle 2,-7,11\rangle$:

$$\langle 2,-7,11\rangle + 5\cdot\frac{1}{5}\langle 3,4,0\rangle = \langle 5,-3,11\rangle$$

Mar 19-12:43 PM

15.7, 15.8

Find distance from pt. to plane.

Find the point that minimizes that distance using

15.7 tech.

15.8 tech.

$\int$ 15.7 #40, 15.8 #28

#39, #27 have answers.

15.7 #30

$P: \quad x - y + z = 4$

$(x-y)^2 = (y-x)^2$

$Q: \quad (1,2,3)$

$$F(x,y,z) = (x-1)^2 + (y-2)^2 + (z-3)^2$$

$$z = -x + y + 4 \Longrightarrow$$

$$F(x,y,z) = f(x,y) = (x-1)^2 + (y-2)^2 + (-x+y+4-3)^2$$
$$= (x-1)^2 + (y-2)^2 + (x-y-1)^2$$

$$f_x = 2(x-1) + 2(x-y-1)$$
$$= 2x - 2 + 2x - 2y - 2$$
$$= 4x - 2y - 4 \overset{\text{SET}}{=} 0$$
$$\Longrightarrow \boxed{2x - y = 2}$$

$$f_y = 2(y-2) + 2(x-y-1)(-1)$$
$$= 2y - 4 + (2x - 2y - 2)(-1)$$
$$= 2y - 4 - 2x + 2y + 2$$
$$= -2x + 4y - 2 \overset{\text{SET}}{=} 0$$
$$\boxed{x - 2y = -1}$$

Solve the system:

$$x - 2y = -1 \Longrightarrow x = 2y - 1$$
$$2x - y = 2$$

$$2(2y-1) - y = 2$$
$$4y - 2 - y = 2$$
$$3y = 4$$
$$\boxed{y = \tfrac{4}{3}}$$

$$x = 2y - 1 = \tfrac{8}{3} - \tfrac{3}{3} = \boxed{\tfrac{5}{3} = x}$$

$$z = \dots = \tfrac{11}{3}$$

$$(x,y,z) = \left(\tfrac{5}{3}, \tfrac{4}{3}, \tfrac{11}{3}\right)$$

$$F(x,y,z) = \overset{\text{Min}}{(x-1)^2 + (y-2)^2 + (z-3)^2}$$

s.t.

$$x - y + z = 4$$

$$G(x,y,z) = x - y + z - 4$$

$$\nabla F = \lambda \nabla G$$

$$\langle 2(x-1),\ 2(y-2),\ 2(z-3) \rangle = \lambda \langle 1, -1, 1 \rangle$$

$$2x - 2 = \lambda \qquad 2y - 4 = -\lambda \qquad 2z - 6 = \lambda$$

$$2x = \lambda + 2 \qquad 2y = \lambda + 4 \qquad 2z = \lambda + 6$$

$$x = \frac{\lambda+2}{2} \qquad y = \frac{-\lambda+4}{2} \qquad z = \frac{\lambda+6}{2}$$

$$x - y + z = 4$$

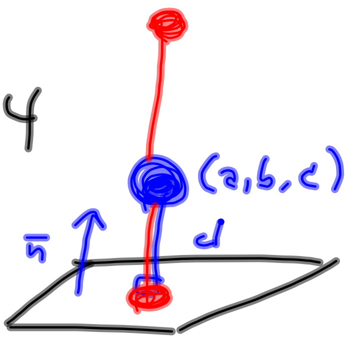
$$\frac{\lambda+2}{2} - \frac{-\lambda+4}{2} + \frac{\lambda+6}{2} = 4$$

$$\lambda + 2 + \lambda - 4 + \lambda + 6 = 8$$

$$3\lambda + 4 = 8$$

$$\lambda = \frac{4}{3}$$

$$x = \frac{6}{2}$$




$$y = \frac{-\lambda+4}{2} = \frac{-\frac{4}{3} + \frac{12}{3}}{2} = \frac{4}{3} = y \qquad z = \frac{11}{3}, \ldots$$

$$x = \frac{\lambda+2}{2} = \frac{\frac{4}{3}+\frac{6}{3}}{2} = \frac{\frac{10}{3}}{2} = \frac{5}{3} = x$$