§ 15.5 coms

$$\frac{\partial}{\partial x}\left[\frac{y}{x}\right] = -\frac{y}{x^2}$$

$$z = \arctan\left(\frac{y}{x}\right)$$

$x = e^{t} \longrightarrow$

$$\frac{dx}{dt} = e^{t}$$

$$\frac{\partial z}{\partial x} = \left(\frac{1}{1+\left(\frac{y}{x}\right)^2}\right)\left(-\frac{y}{x^2}\right)\left(e^{t}\right)$$

$y = 1 - e^{-t}$

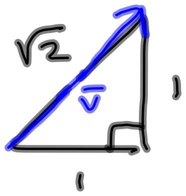
STOP RIGHT THERE

$$\frac{dz}{dt} = \left(\frac{1}{1+\left(\frac{y}{x}\right)^2}\right)\left(-\frac{y}{x^2}\right)\left(e^{t}\right) + \frac{\frac{1}{x}}{1+\left(\frac{y}{x}\right)^2}e^{-t}$$

§ 15.6

(4) $f(x,y) = x^2y^3 - y^4$ (a) $(2,1)$ in

direction of $\theta = \frac{\pi}{4}$



$\bar{u} = \frac{\bar{v}}{|\bar{v}|} = \frac{\langle 1,1 \rangle}{\sqrt{1^2+1^2}}$

$= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$\bar{u} = \langle \cos\frac{\pi}{4}, \sin\frac{\pi}{4} \rangle$

$= \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

is fine, but maybe thinking too hard.

$\nabla f(1,2) \cdot \bar{u}$

$=$

$\nabla f = \langle 2xy^3, 3x^2y^2 - 4y^3 \rangle$

$\nabla f(2,1) = \langle 4, 8 \rangle$

$\nabla f(2,1) \cdot \bar{u} = \langle 4, 8 \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

$= \frac{4}{\sqrt{2}} + \frac{8}{\sqrt{2}}$

$= \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$

#40

Plane $F_x(x-x_0)+F_y(y-y_0)+F_z(z-z_0)=0$

$\Longrightarrow \quad 8(x-4)-1(y+7)-6(z-3)=0$

$\bar{n} = \langle 8,-1,-6\rangle$ is parallel to $\langle \frac{8}{6}, -\frac{1}{6}, -\frac{6}{6}\rangle$

$x = x_0 + at$
$y = y_0 + bt$
$z = z_0 + ct$

where $\langle a,b,c\rangle$ is direction vector for the line.

$$\boxed{\begin{aligned} x &= 4+8t \\ y &= -7-t \\ z &= 3-6t \end{aligned}}$$

$x = 4 + \frac{4}{3}t$
$y = -7 - \frac{1}{6}t$ is fine.
$z = 3 - t$

$$\nabla f = \lambda \nabla g$$

$f(x,y)$ to be optimized
subject to $g(x,y) = K$
when f is optimized, then its level curves are parallel to the constraint

$\underbrace{\text{parallel}}_{\text{tangent.}}$

See § 15.8 #2

$z = f(x,y) = x^2 + y$
s.t.
$x^2 + y^2 = 1$

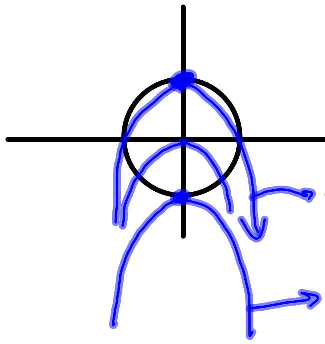
$x^2 + y = 1$ → level curve for $f(x,y)$

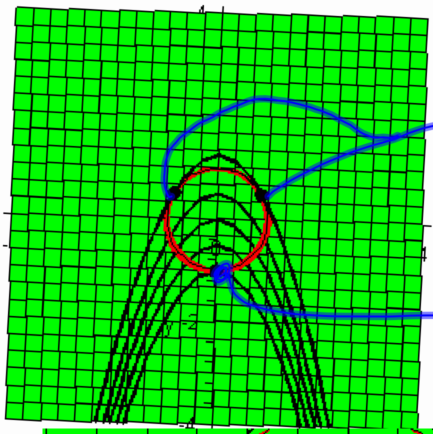
$y = 1 - x^2$

$x^2 + y = -1$

$y = -1 - x^2$

Looking Down
"Plan View"

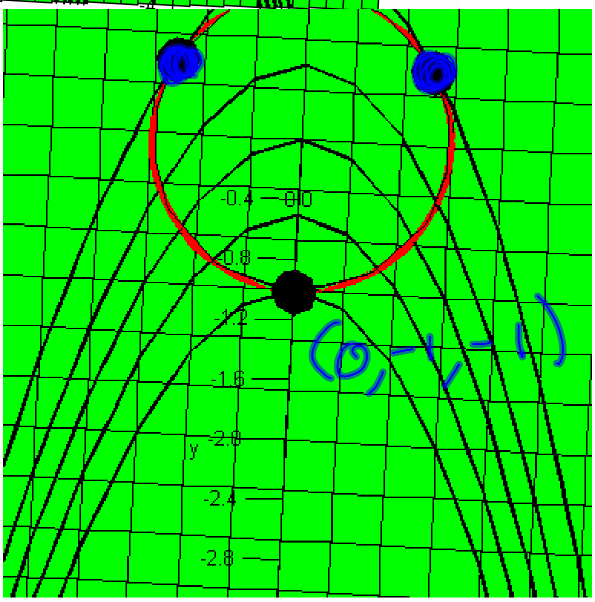




The two max points.

The low point

(



$f(0,-1)$

$= 0^2 - 1 = -1$

$\leadsto (0,-1,-1)$

See? Constraint & level curves are tangent @ max/min.

That's the meaning of $\nabla f = \lambda \nabla g$

$\langle 1,1,3\rangle$ is parallel to

$\langle .5,.5,1.5\rangle$, because, $\lambda = 2$

gives

$\bar{u} = 2\bar{v}$

$\langle 1,1,3\rangle = 2\langle .5,.5,1.5\rangle$

$f(x,y) = e^{xy}$ s.t. $x^3 + y^3 = 16$

$g(x,y) = x^3 + y^3$

$f_x = ye^{xy}$

$g_x = 3x^2$

$f_y = xe^{xy}$

$g_y = 3y^2$

Lagrange

$\underbrace{ye^{xy}} = \lambda \underbrace{3x^2}$ & $xe^{xy} = 3\lambda y^2$

Note: $x=0 \Rightarrow y=0$ ✗

can't happen.

Then $x^3 + y^3 \neq 16$

$ye^{xy} = \lambda 3x^2$ & $xe^{xy} = 3\lambda y^2$

$\lambda = 0$
Nevah!

$\lambda = \frac{ye^{xy}}{3x^2}$ $\lambda = \frac{xe^{xy}}{3y^2}$

$\frac{y\cancel{e^{xy}}}{\cancel{3}x^2} = \frac{x\cancel{e^{xy}}}{\cancel{3}y^2} \Rightarrow y^3 = x^3$

$\Rightarrow x = y$

So $x^3 + y^3 = 2x^3 = 16$

$x^3 = 8$

$x = 2 = y$

Max @ $(2, 2, e^4)$

$\nexists$ min, since we can make $x^3+y^3=16$ & never make $e^{xy}=0$, but we can make it close.

why?