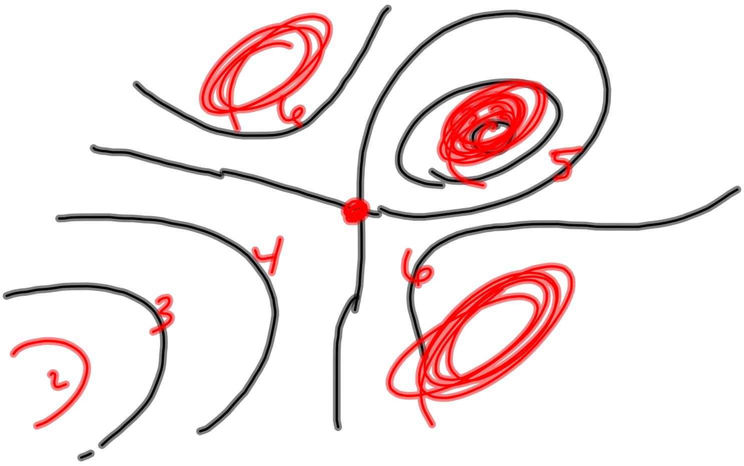
§15.7 questions?

③



$$\boxed{\nabla f(x,y) = \lambda \nabla g(x,y)}$$

f & g are ∥ when they touch

Maximize $f(x,y) = x^2 + y$

s.t. $[\,x^2+y^2=1\,]$

Now, $g(x,y) = x^2+y^2-1=0$

$\nabla f = \langle f_x, f_y \rangle = \langle 2x, 1 \rangle$

$\nabla g = \langle 2x, 2y \rangle$

$\langle 2x, 1 \rangle = \lambda \langle 2x, 2y \rangle$ for some (x,y,λ) when we're in the right spot(s).

$2x = \lambda 2x$ $\quad \lambda = 1$ OR $x = 0$

$1 = 2\lambda y$

$y = \frac{1}{2\lambda}$

$\lambda = 1 \Rightarrow y = \frac{1}{2}$

Plug it into the constraint

$x^2+y^2=1$

$x^2 + (\frac{1}{2})^2 = 1$

$x^2 + \frac{1}{4} = 1$

$x^2 = \frac{3}{4}$

$x = \pm \frac{\sqrt{3}}{2}$

Send THIS to $f(x,y)$

$\boxed{f(\frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{3}{4} + \frac{1}{2} = \frac{5}{4}}$

$\boxed{f(-\frac{\sqrt{3}}{2}, \frac{1}{2}) = \frac{5}{4}}$

$x=0$:

$x^2+y^2 = 1$

$y^2 = 1$

$x^2+y=f(x,y)$ $\quad y = \pm 1$

$f(0,1) = 0^2+1 = 1$ $\quad (x, f(x))$

$f(0,-1) = 0^2 - 1 = -1$

$(0,1,1)$

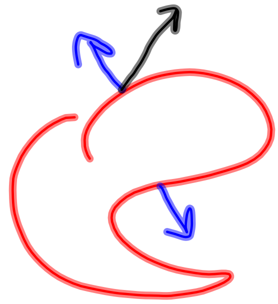
$(0,1,-1)$ Min $\quad (x,y,f(x,y))$

$(\pm \frac{\sqrt{3}}{2}, \frac{5}{4})$ Max

z

This was §15.8 #2b.

#2a tries to get you into the spirit of the theory.



#10 $f(x,y,z) = x^2y^2z^2$

S.t. $x^2+y^2+z^2=1$

$g(x,y) = x^2+y^2+z^2-1$

$f_x = \lambda g_x$ $\quad$ $f_y = \lambda g_y$ $\quad$ $f_z = \lambda g_y$

$2xy^2z^2 = 2\lambda x$ $\quad$ $2yx^2z^2 = 2\lambda y$ $\quad$ $2zx^2y^2 = 2\lambda z$

$y^2z^2 = \lambda$ $\quad$ $x^2z^2 = \lambda$ $\quad$ $x^2y^2 = \lambda$

$\therefore\ x^2y^2 = y^2z^2 = x^2z^2$

$x^2y^2 = x^2z^2$

$y^2 = z^2$

$\ldots = x^2$

(1) $\lambda \neq 0$

$x^2+y^2+z^2$

$=3x^2 = 1$

$\Longrightarrow x^2 = \frac{1}{3}$

$x = \pm\frac{1}{\sqrt{3}}$

$y = \pm\frac{1}{\sqrt{3}}$

$z = \pm\frac{1}{\sqrt{3}}$

$\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$ is candidate.

$\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ .. another.

$f = \frac{1}{27}$ @ each of these.

$f(x,y,z) = x^2y^2z^2$

$\lambda = 0$

$\lambda = 0$:

$y^2z^2 = \lambda = 0$ $\quad x^2z^2 = \lambda$ $\quad x^2y^2 = \lambda$

$y = 0$ or $z = 0$

$y=0$: $\quad z = 0$ $\quad y = 0$

$x^2+y^2+z^2 = 1$ $\quad x^2+y^2=1$ $\quad x^2+z^2=1$

$x^2+z^2 = 1$ $\quad$ Good Start