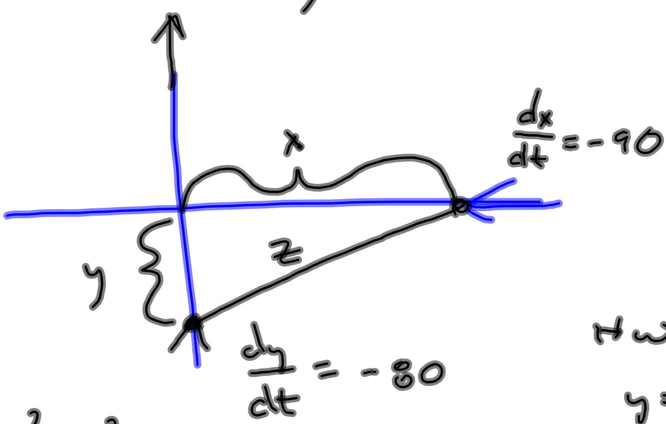
§15.5 Monday

§15.6 Tuesday




when $x = .3$, $y = .4$, $z = \sqrt{.25} = .5$

$$z^2 = x^2 + y^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2z} \quad \text{etc.}$$

§15.6 #s 4, 8, 9, 16, 19, 24, 34, 40, 52

45, 46 Good Maple Projects.

§15.7 #s 3, 6, 11, 19, 21, 26, 30, 33, 39, 40

§15.8 #s 2, 3, 6, 10, 13, 14, 27, 28, 41, 45*, 46*

#46 is Cauchy-Schwarz inequality

Integral Estimates.

$$\sum a_i b_i \leq \sqrt{\sum a_i^2}\sqrt{\sum b_i^2}$$

$$\int f(x)g(x)\,dx \leq \sqrt{\int f(x)^2\,dx}\sqrt{\int g(x)^2\,dx}$$

which generalizes to Hölder:

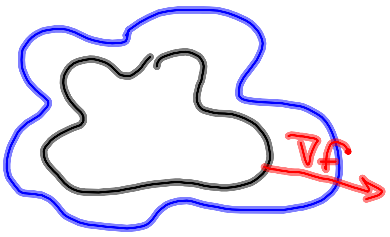
$$\int f(x)g(x)\,dx \leq \sqrt[p]{\int f(x)^p\,dx}\ \sqrt[q]{\int g(x)^q\,dx}$$

where $\frac{1}{p} + \frac{1}{q} = 1$

Mar 2-12:13 PM

Recall $\nabla f$ is $\perp$ to the level curves

$z = f(x,y)$ has level curves $f(x,y) = K$



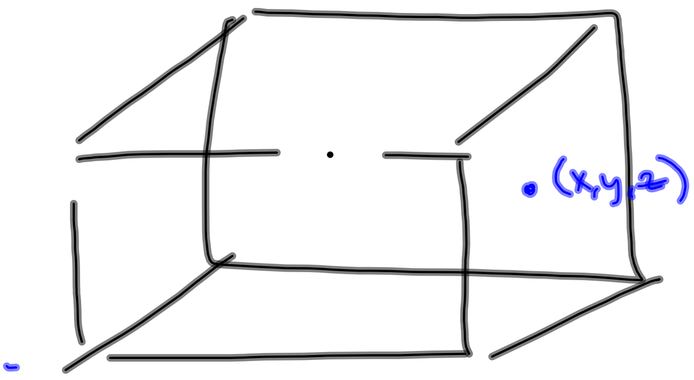

$\nabla f = \langle f_x, f_y \rangle$

$\nabla f$ is $\perp$ to level SURFACES !

$$\boxed{f(x,y,z) = K}$$

$\nabla f = \langle f_x, f_y, f_z \rangle$ is orthogonal to the level SURFACE.

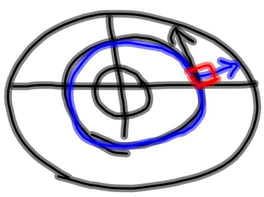
See pg 953. Fig 9.




$F(x,y,z)$ is a number.

All points yielding $F(x,y,z) = 7$ comprise a level SURFACE.

$f(x,y) = x^2+y^2$

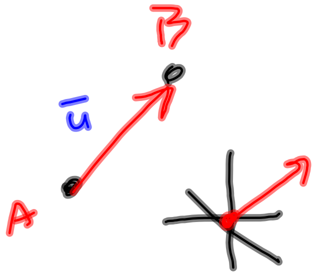


tangent to the contour is $\perp$ to $\nabla f$.

Tangent Plane is orthogonal to the gradient of the level SURFACE.

Let $A(x_0, y_0, z_0)$ be a point on the plane.

Then let $B(x, y, z)$ be another point on the plane.




Then $\langle x-x_0, y-y_0, z-z_0 \rangle$ is a vector $\bar{u}$ that's parallel to the plane.

So, it's $\perp$ to $\nabla f$.

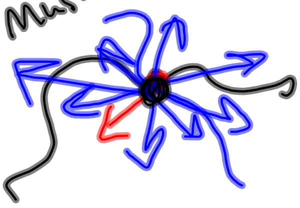
So $\nabla f \cdot \bar{u} = 0$

$$\langle f_x, f_y, f_z \rangle \cdot \langle x-x_0, y-y_0, z-z_0 \rangle = 0$$

$$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$$

is the equation of the tangent plane to the level surface!

Pg 953 Must Read.



Normal Line @ $(x_0, y_0, z_0)$

is
$$x = x_0 + f_x t$$
$$y = y_0 + f_y t$$
$$z = z_0 + f_z t$$

40 $\boxed{y = x^2 - z^2}$ Find eq'n of normal plane to this surface @ (4,7,3)

Old Way

$z^2 = x^2 - y$

$z = \pm\sqrt{x^2-y}$

$f(x,y) = +\sqrt{x^2-y}$ is the piece we're on, since $z=3$, $x=4$, $y=7$

$z = f_x(x-x_0) + f_y(y-y_0) + z_0$

$f_x = \frac{1}{2}(x^2-y)^{-\frac{1}{2}}(2x) = \dfrac{x}{\sqrt{x^2-y}}$

$f_x(4,7) = \dfrac{4}{\sqrt{16-7}} = \dfrac{4}{3}$

$f_y = \frac{1}{2}(x^2-y)^{-\frac{1}{2}}(-1) = -\dfrac{1}{2\sqrt{x^2-y}}$

$f_y(4,7) = -\dfrac{1}{2\cdot 3}$

$z = \frac{4}{3}(x-4) - \frac{1}{6}(y-7) + 3$

$\frac{4}{3}(x-4) - \frac{1}{}(y-7) + 3 - z = 0$

$\frac{4}{3}(x-4) - \frac{1}{6}(y-7) - 1(z-3) = 0$

$\boxed{\bar{\omega} = \langle \frac{4}{3}, -\frac{1}{6}, -1 \rangle}$ is normal to tangent plane.

So
$x = 4 + \frac{4}{3}t$
$y = 7 - \frac{1}{6}t$
$z = 3 - 1t$
Normal Line.

$\bar{n} = \langle 8, -1, -6 \rangle$ is be the

$8(x-4) - 1(y-7) - 6(z-3) = 0$

New Way

$x^2 - y - z^2 = 0$

$F(x,y,z) = 0$

$\nabla F = \langle 2x, -1, -2z \rangle$

$\nabla F(4,7,3) = \langle 8, -1, -6 \rangle$

Tan Plane:

$8(x-4) - 1(y-7) - 6(z-3) = 0$

Normal Line:

$x = 4 + 8t$
$y = 7 - t$
$z = 3 - 6t$

$x = 4 + \frac{4}{3}t$
$y = 7 - \frac{1}{6}t$
$z = 3 - 1t$
Normal Line.

(12) $f(x,y) = \ln(x^2+y^2)$ @ $(2,1)$

in direction of $\bar{u} = \langle -1, 2 \rangle$

$$D_{\bar{u}} = \nabla f \cdot \frac{\bar{u}}{|\bar{u}|}$$

$\bar{u}$ isn't a unit vector!

Make one!

$$|\bar{u}| = \sqrt{1^2+2^2} = \sqrt{5}$$

$$D_{\bar{u}} = \nabla f \cdot \frac{\langle 1,2 \rangle}{\sqrt{5}}$$

$$\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$= \frac{1}{\sqrt{5}} \nabla f \cdot \langle 1,2 \rangle$$

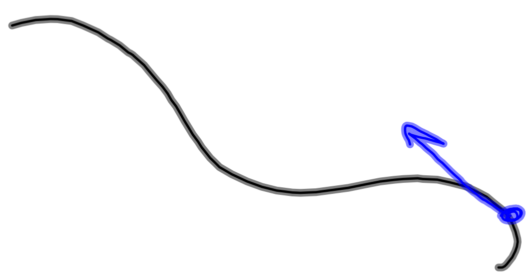
$$= \frac{1}{\sqrt{5}} \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \right\rangle \cdot \langle -1, 2 \rangle$$

@ $(2,1)$:

$$= \frac{1}{\sqrt{5}} \left\langle \frac{4}{5}, \frac{2}{5} \right\rangle \cdot \langle -1, 2 \rangle$$

$$= \frac{1}{\sqrt{5}} \left( -\frac{4}{5} + \frac{4}{5} \right) = 0 \; !$$

Wha' happink?



$\langle -1, 2 \rangle$ happens to be tangent to the level curve

§15.6 Due Mañana

§15.7 Due Wed.

§15.8 Fri.

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§15.7 #44 Minimize $x^2+y^2+z^2$
s.t. $x+y+z=12$

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Critical Point: $f_y = f_x = 0$
(OR Both undefined) We'll keep it smooth

$f^*(x,y,z) = x^2+y^2+z^2$ But look: The constraint lets us solve for z & substitute in to $f^*(x,y,z)$

$x+y+z=12 \implies$

$z = 12-x-y \implies$

$$f(x,y) = x^2 + y^2 + (12-x-y)^2$$

$$f_x = 2x + 2(12-x-y)(-1) = 2x - 24 + 2x + 2y$$
$$= 4x + 2y - 24 \overset{\text{SET}}{=} 0$$

$$f_y = 2y + 2(12-x-y)(-1)$$
$$= 2y - 24 + 2x + 2y$$
$$= 4y + 2x - 24 \overset{\text{SET}}{=} 0$$

Want $(x,y) \ni$ Both eq'ns are true

$$4x + 2y - 24 = 0 \qquad 2x + y - 12 = 0$$
$$2x + 4y - 24 = 0 \longrightarrow x + 2y - 12 = 0$$

$$x + 2y - 12 = 0 \qquad -2x - 4y + 24 = 0$$
$$2x + y - 12 = 0 \qquad 2x + y - 12 = 0$$
$$-3y + 12 = 0$$
$$-3y = -12$$
$$y = 4$$

$$x + 2(4) - 12 = 0$$
$$x + 8 - 12 = 0$$
$$x - 4 = 0$$
$$x = 4$$

$$(x,y) = (4,4)$$

Mar 5-12:08 PM

Now $(x,y)=(4,4)$ is candidate. But we have a new $2^{nd}$ derivative test for surfaces. To minimize $f(x,y) \ni g(x,y)=K$

$f_x = f_y = 0$

T3 §15.7

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$D(a,b) > 0$ & $f_{xx}(a,b) > 0 \Longrightarrow f(a,b)$ is min.

$D(a,b) > 0$ & $f_{xx}(a,b) < 0 \Longrightarrow f(a,b)$ is max.

$D(a,b) < 0$ Neither

$D(a,b) = 0$ we don't know.

$f_x = 2x + 2(12-x-y)(-1) = \underline{4x+2y-24}$ Clairaut ?

$f_{xx} = 4$, $f_{xy} = 2$

$f_y = 2y + 2(12-x-y)(-1) = 2x+4y-24$

$f_{yy} = 4$, $f_{yx} = 2 = \frac{\partial}{\partial x}\left[\frac{df}{dy}\right]$

$$D(a,b) = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} = \underline{16-4 = 12 > 0}$$

So $f(x,y)$ is local min.

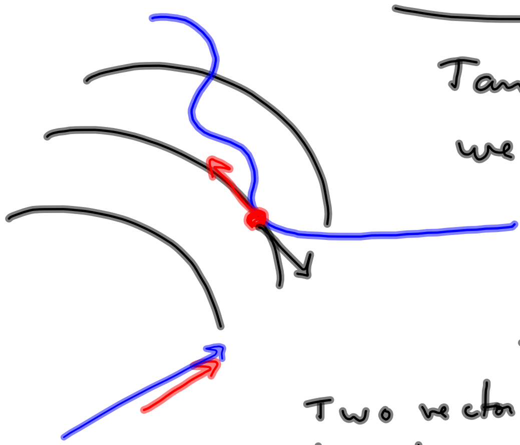
$z = 12-x-y = 12-8 = 4 \Longrightarrow$ $(4,4,4)$ minimizes the sum of the squares.

Mar 5-12:35 PM

§15.8 #32 Re-work §15.7 #44 with Lagrange Multipliers.

Minimize $F(x,y,z) = x^2+y^2+z^2$

s.t. $G(x,y,z) = x+y+z=12$ Level surface



Tangents parallel when we're optimized

$$\nabla F = \lambda \nabla G$$

for some $\lambda \neq 0$

Two vectors are parallel when they're multiples of each other

~~$\bar{n} = \langle 2,6,8 \rangle$~~

$\bar{n} = \langle 1,3,4 \rangle$

$\nabla F = \langle 2x, 2y, 2z \rangle$

$\lambda \nabla G = \lambda \langle 1,1,1 \rangle$

Want $x, y, z$ & $\lambda$ so that

$2x = \lambda$

$2y = \lambda$

$2z = \lambda$

$x = \frac{\lambda}{2} = y = z$

$\nabla F = \lambda \nabla G$

$\langle 2x, 2y, 2z \rangle = \lambda \langle 1,1,1 \rangle$

$x = y = z$ ✓

So $x+y+z=12$

becomes $x+x+x = 12$

$3x = 12$

$x = 4 = y = z$

So $(x,y,z) = (4,4,4)$ satisfies us

§15.6 # 34

meters $z = 1000 - .005x^2 - .01y^2 = f(x,y)$

$(x_0, y_0, z_0) = (60, 40, 966)$

If you walk South, are you ascending or descending and @ what rate if pos. x-direction is East & pos. y- " " South

$\bar{u} = \langle 0, -1 \rangle$ " no

$|\bar{u}| = \sqrt{1^2} = 1$

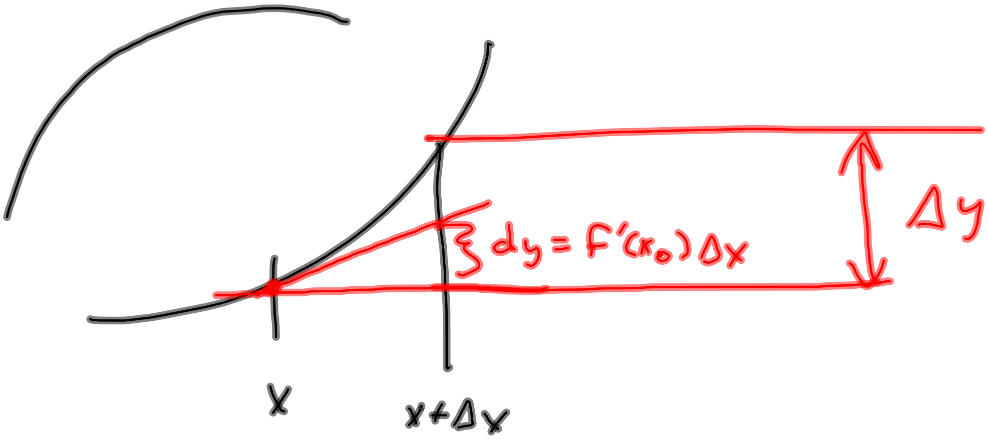
$\nabla f = \langle -.01x, -.02y \rangle$

$D_{\bar{u}} = \langle -.01x, -.02y \rangle \cdot \langle 0, -1 \rangle$

$= .02y$

@ $(60, 40, 966)$, we have

$D_{\bar{u}} = (.02)(40) = \dfrac{.8 \text{ m}}{1 \text{ m}}$ up South.




$$\frac{\Delta y}{\Delta x} \approx f'(x)$$

$$\frac{\Delta y}{\Delta x} - f'(x) = \text{small} = \varepsilon$$

$$\Delta y - f'(x)\Delta x = \varepsilon \Delta x$$

$$\Delta y = f'(x)\Delta x + \varepsilon \Delta x$$

$$f(x_0+\Delta x) - f(x_0) = \Delta y$$

Mar 5-1:11 PM