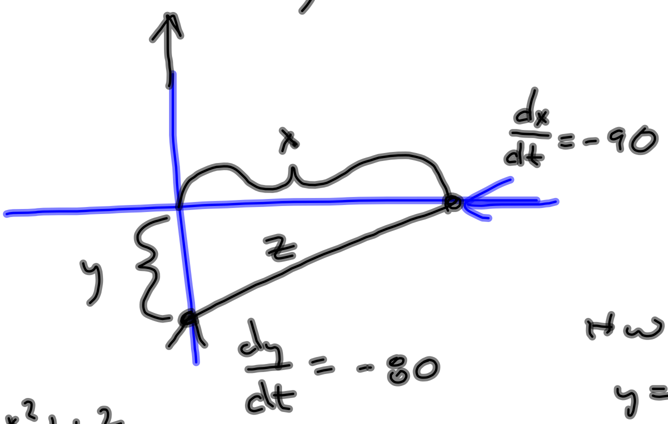
§15.5 Monday

§15.6 Tuesday



when $x = .3$, $y = .4$, $z = \sqrt{.25} = .5$

$$z^2 = x^2 + y^2$$

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\Rightarrow \frac{dz}{dt} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2z} \quad \text{etc.}$$

Mar 2-12:04 PM

§15.6 #s 4, 8, 9, 16, 19, 24, 34, 40, 52

45, 46 → Good Maple Projects.

§15.7 #s 3, 6, 11, 19, 21, 26, 30, 33, 39, 40

§15.8 #s 2, 3, 6, 10, 13, 14, 27, 28, 41, 45*, 46*

#46 is Cauchy-Schwarz inequality

Integral Estimates.

$$\sum a_i b_i \le \sqrt{\sum a_i^2}\sqrt{\sum b_i^2}$$

$$\int f(x)g(x)\,dx \le \sqrt{\int f(x)^2\,dx}\sqrt{\int g(x)^2\,dx}$$

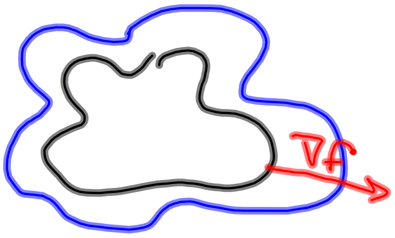
which generalizes to Hölder:

$$\int f(x)g(x)\,dx \le \sqrt[p]{\int f(x)^p\,dx}\sqrt[q]{\int g(x)^q\,dx}$$

where $\frac{1}{p} + \frac{1}{q} = 1$

Recall ∇f is $\perp$ to the level curves

$z = f(x,y)$ has level curves
$f(x,y) = K$



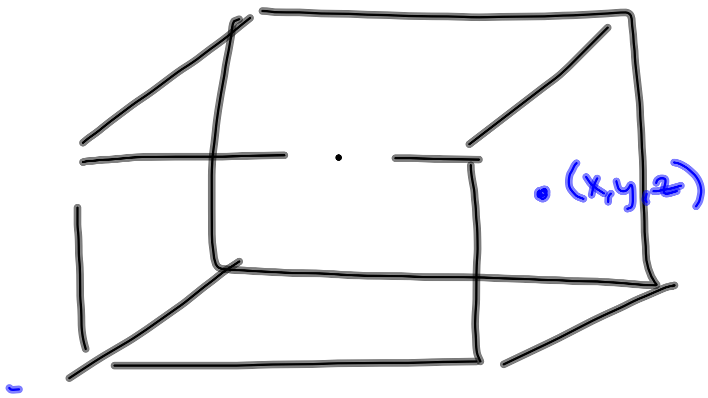
$\nabla f = \langle f_x, f_y \rangle$

∇f is $\perp$ to Level SURFACES !

$$\boxed{f(x,y,z) = K}$$

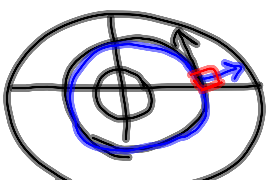
$\nabla f = \langle f_x, f_y, f_z \rangle$ is orthogonal to the level SURFACE.

See pg 953. Fig 9.



$F(x,y,z)$ is a number.
All points yielding
$F(x,y,z) = 7$
comprise a level SURFACE.

$f(x,y) = x^2 + y^2$

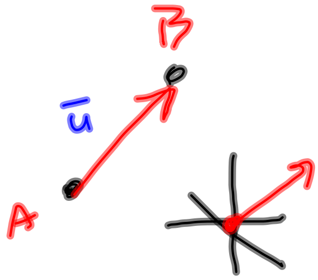


tangent to the contour is $\perp$ to ∇f.

Tangent Plane is orthogonal to the gradient of the level SURFACE.

Let A (x_0, y_0, z_0) be a point on the plane.

Then let B (x,y,z) be another point on the plane.



Then $\langle x-x_0, y-y_0, z-z_0 \rangle$ is a vector $\bar{u}$ that's parallel to the plane.

So, it's $\perp$ to ∇f.

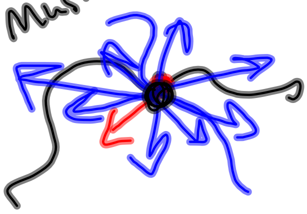
So $\nabla f \cdot \bar{u} = 0$

$$\langle f_x, f_y, f_z \rangle \cdot \langle x-x_0, y-y_0, z-z_0 \rangle = 0$$

$$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$$

is the equation of the tangent plane to the level surface!

Pg 953 Must Read.



Normal Line @ (x_0, y_0, z_0) is

$$x = x_0 + f_x t$$
$$y = y_0 + f_y t$$
$$z = z_0 + f_z t$$

(40) $y = x^2 - z^2$

Find eq'n of normal plane to this surface @ (4,7,3)

Old Way

$z^2 = x^2 - y$

$z = \pm\sqrt{x^2-y}$

$f(x,y) = +\sqrt{x^2-y}$ is the piece we're on, since (z=3), x=4, y=7

$z = f_x(x-x_0) + f_y(y-y_0) + z_0$

$f_x = \frac{1}{2}(x^2-y)^{-\frac{1}{2}}(2x) = \frac{x}{\sqrt{x^2-y}}$

$f_x(4,7) = \frac{4}{\sqrt{16-7}} = \frac{4}{3}$

$f_y = \frac{1}{2}(x^2-y)^{-\frac{1}{2}}(-1) = -\frac{1}{2\sqrt{x^2-y}}$

$f_y(4,7) = -\frac{1}{2 \cdot 3}$

$z = \frac{4}{3}(x-4) - \frac{1}{6}(y-7) + 3$

$\frac{4}{3}(x-4) - \frac{1}{6}(y-7) + 3 - z = 0$

$\frac{4}{3}(x-4) - \frac{1}{6}(y-7) - 1(z-3) = 0$

$\bar{w} = \langle \frac{4}{3}, -\frac{1}{6}, -1 \rangle$ is normal to tangent plane.

So
$x = 4 + \frac{4}{3}t$
$y = 7 - \frac{1}{6}t$
$z = 3 - 1t$
} Normal Line.

$\bar{n} = \langle 8, -1, -6 \rangle$ is better

$8(x-4) - 1(y-7) - 6(z-3) = 0$

New Way

$x^2 - y - z^2 = 0$

$F(x,y,z) = 0$

$\nabla F = \langle 2x, -1, -2z \rangle$

$\nabla F(4,7,3) = \langle 8, -1, -6 \rangle$

Tan Plane:

$8(x-4) - 1(y-7) - 6(z-3) = 0$

Normal Line:

$x = 4 + 8t$
$y = 7 - t$
$z = 3 - 6t$

$x = 4 + \frac{4}{3}t$
$y = 7 - \frac{1}{6}t$
$z = 3 - 1t$
} Normal Line.

(12) $f(x,y) = \ln(x^2+y^2)$ @ $(2,1)$

in direction of $\bar{u} = \langle -1, 2 \rangle$

$$D_{\bar{u}} = \nabla f \cdot \frac{\bar{u}}{|\bar{u}|}$$

$\bar{u}$ isn't a unit vector! Make one!

$$|\bar{u}| = \sqrt{1^2+2^2} = \sqrt{5}$$

$$D_{\bar{u}} = \nabla f \cdot \frac{\langle 1,2 \rangle}{\sqrt{5}}$$

$$\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$= \frac{1}{\sqrt{5}} \nabla f \cdot \langle 1,2 \rangle$$

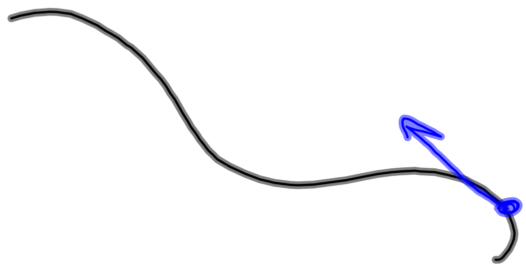
$$= \frac{1}{\sqrt{5}} \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \right\rangle \cdot \langle -1, 2 \rangle$$

@ $(2,1)$:

$$= \frac{1}{\sqrt{5}} \left\langle \frac{4}{5}, \frac{2}{5} \right\rangle \cdot \langle -1, 2 \rangle$$

$$= \frac{1}{\sqrt{5}} \left(-\frac{4}{5} + \frac{4}{5} \right) = 0 \; !$$

Wha' happink?



$\langle -1,2 \rangle$ happens to be tangent to the level curve