(27) $\sqrt{xy} = 1 + x^2 y$ $\qquad \dfrac{dy}{dx} = \dfrac{-F_x}{F_y}$

Find $\frac{dy}{dx}$: $\frac{1}{2}(xy)^{-\frac{1}{2}}(y + xy') = 2xy + x^2 y'$

$$\frac{1}{2}(xy)^{-\frac{1}{2}}y + \frac{1}{2}(xy)^{-\frac{1}{2}}(xy') - 2xy - x^2 y' = 0$$

$$\frac{1}{2}(xy)^{-\frac{1}{2}}xy' - x^2 y' = 2xy - \frac{1}{2}(xy)^{-\frac{1}{2}}y$$

Factor out y'

divide $\longrightarrow$ $\quad y' = \dfrac{2xy - \frac{1}{2}(xy)^{-\frac{1}{2}}y}{\frac{1}{2}(xy)^{-\frac{1}{2}}x - x^2}$

(27) $\sqrt{xy} = 1 + x^2y$ $\qquad \frac{dy}{dx} = \frac{-F_x}{F_y}$

Find $\frac{dy}{dx}$: $\underbrace{\sqrt{xy} - 1 - x^2y}_{F(x,y)} = 0$

$F_x = \frac{1}{2}(xy)^{-\frac{1}{2}}(y) - 2xy$

$F_y = \frac{1}{2}(xy)^{-\frac{1}{2}}(x) - x^2$

Lot Less work But formulaic.

$\Rightarrow \frac{dy}{dx} = -\frac{\frac{1}{2}(xy)^{-\frac{1}{2}}y - 2xy}{\frac{1}{2}(xy)^{-\frac{1}{2}}x - x^2}$

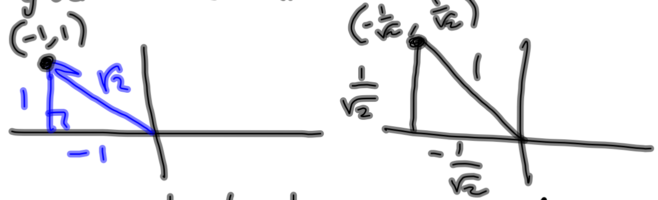
$$\frac{2xy - \frac{1}{2}(xy)^{-\frac{1}{2}}y}{\frac{1}{2}(xy)^{-\frac{1}{2}}x - x^2}$$

§ 15.1-15.3 Solns posted.
15.4 after class, today.

§15.6 Gradient & Directional Derivatives

$$f_x(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+h,y) - f(x_0,f(y_0))}{h}$$

Suppose you're moving Northwest and want to figure the steepness in that direction.

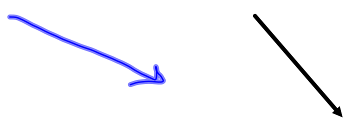


Unit vector headed Northwest : $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

This is the idea for § 15.6.

$\boxed{D}$ Directional Derivative of f @ (x_0,y_0) in the direction of $\bar{u} = \langle u_1, u_2 \rangle$ is given by

$$\lim_{h\to 0} \frac{f(x_0+u_1h, y_0+u_2h) - f(x_0,y_0)}{h} = D_u(f)$$



we've already done those for

$\bar{u} = \langle 1, 0 \rangle = \bar{i}$

$\bar{w} = \langle 0, 1 \rangle = \bar{j}$

$$\lim_{h\to 0} \frac{f(x_0+1h, y_0+0h) - f(x_0,y_0)}{h} = f_x = D_i(f)$$

$$f_y = \frac{f(x_0+0h, y_0+1h) - f(x_0,y_0)}{h} = D_j(f)$$

$\boxed{D}$ Gradient of $f(x,y)$ is

$$\nabla f = \langle f_x, f_y \rangle$$

$\boxed{T3}$ If f is dif$b^{\underline{l}}$ then f has directional derivative $D_{\bar{u}}$ for ANY unit vector $\bar{u} = \langle a, b \rangle$ and it's given by

$$f_x a + f_y b = \langle f_x, f_y \rangle \cdot \langle a, b \rangle = \nabla f \cdot \bar{u}$$

$\langle a, b \rangle = \langle 1, 0 \rangle a + \langle 0, 1 \rangle b$ is the idea.

$$\frac{f(x_0+ha, y_0+hb) - f(x_0, y_0)}{h}$$

Analyst's trick

$$= \frac{f(x_0+ha, y_0+hb) \boxed{- f(x_0+ha, y_0) + f(x_0+ha, y_0)} - f(x_0, y_0)}{h}$$

$$= \frac{1}{h}\left[f(x_0+ha, y_0+hb) - f(x_0+ha, y_0)\right] + \frac{1}{h}\left[\frac{f(x_0+ha, y_0) - f(x_0, y_0)}{1}\right]$$

$$= \frac{b}{h}\left[\frac{f(x_0+ha, y_0+hb) - f(x_0+ha, y_0)}{b}\right] + a\left[\frac{f(\ldots\ldots \text{ same}}{ah}\right]$$

$$= \boxed{\left(\frac{f(x_0+ha, y_0+hb) - f(x_0+ha, y_0)}{bh}\right) b} + \underbrace{- \; - \; - \; -}_{f_x a}$$

$h \to 0$ is same as $bh \to 0$, now

$$\xrightarrow{h \to 0} \boxed{f_y b}$$

$$\bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$$

$$\text{comp}_{\bar{u}} \bar{v} = \frac{\bar{v} \cdot \bar{u}}{|\bar{u}|}$$

$$\text{comp}_{\bar{u}} \nabla f = \frac{\nabla f \cdot \bar{u}}{|\bar{u}|} = \nabla f \cdot \bar{u}$$ is the scalar projection of ∇f on $\bar{u}$

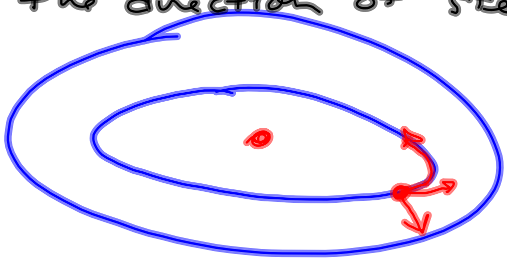
of $\bar{u}$ on ∇f

$\bar{u}$ is unit length:

Maximizing Directional Derivatives.

In what direction is f changing most rapidly?

Gradient is in the direction of steepest descent.



$F(x,y,z)=K$ is a level surface.

Equation of tangent plane to the level surface (Not tangent to the hypersurface associated with $F(x,y,z)$

A solid, embedded in 4-space.

Enrichment.

Hyper cool

Space-time

(x,y,z,t)