§15.4 Tangent Planes

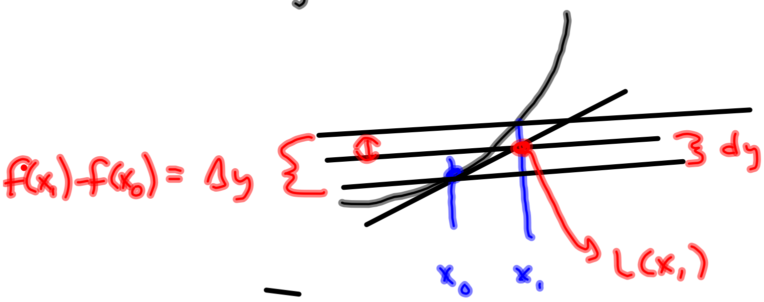
Tangent line @ (x_0, y_0)

$$L(x) = f'(x_0)(x - x_0) + y_0$$

$$y = \cancel{m(x-x_0) + y_0}$$

(x_0, y_0)

$\rightarrow (x_1, \underbrace{y_0 + f'(x_0)(x - x_0)})$



$dy = f'(x)(\underline{x_1 - x_0}) = f'(x)\underline{\Delta x}$
$= f'(x)\underline{dx}$

Differentials:

$$\Delta y \approx dy = f'(x)dx = f'(x)\Delta x$$

$$\Delta y = f'(x)\Delta x + \varepsilon_1 \Delta x$$

In 3-D, tangent PLANES.

$$L(x,y) = f_x(x - x_0) + f_y(y - y_0) + z_0$$

(x_0, y_0, z_0) to the next point

(x_1, y_1, z_1)?

view $z = f(x,y)$ is function of (x,y)

New z-value obtained from old by adding its change in x-direction to its change in y-direction

Feb 28-12:09 PM

2-D

$$\Delta y \approx f'(x)\Delta x = f'(x)dx = f'(x)(x_1 - x_0)$$

3-D

$$\Delta z \approx f_x \Delta x + f_y \Delta y = f_x dx + f_y dy$$

Approximate the error in multiplying 2 numbers together, if the error in each number is $\pm .1$

$$z = xy = f(x,y)$$

$$\Delta z \approx f_x \Delta x + f_y \Delta y$$

$$\approx (y)(.1) + (x)(.1)$$

so the error in multiplying $(5)(6) = 30$ would be

$$(6)(.1) + (5)(.1)$$

$$.6 + .5 = 1.1$$

Estimating paint volume.

Estimating error in a volume measurement.

§ 15.5 #s 4, 7, 25, 28, 32, 40, 35, 42, 45, 48

In Class

deign

∂ = del

§ 15.5 Chain Rule.

Recall:

$$\Delta z = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \varepsilon_1 \Delta x + \varepsilon_2 \Delta y$$

$$\frac{\Delta z}{\Delta t} = \frac{\partial f}{\partial x}\cdot\frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y}\frac{\Delta y}{\Delta t} + \varepsilon_1\frac{\Delta x}{\Delta t} + \varepsilon_2\frac{\Delta y}{\Delta t}$$

$$\xrightarrow{\Delta t \to 0} \boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}}$$

(Makes $\varepsilon_1, \varepsilon_2 \to 0$)

Eq'n $\boxed{2}$ § 15.5

$$\frac{df}{dt} = \frac{df}{dx}\cdot\frac{dx}{dt}, \text{ when } f = f(x)$$

what if $z = f(x,y)$ &

$$x = x(s,t)$$
$$y = y(s,t)$$

$$z = f(x(s,t), y(s,t))$$

makes z a function $g(s,t)$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s}$$

$\frac{\partial z}{\partial t}$ similarly defined.

$\boxed{\text{Eg'n 4}}$ §15.5

$$z = f(x_1, \dots, x_n)$$
$$x_1 = x_1(t_1, \dots, t_m)$$
$$\vdots$$
$$x_n = x_n(t_1, \dots, t_m)$$

$$\frac{\partial z}{\partial t_i} = \sum_{k=1}^{n} \frac{\partial z}{\partial x_k}\cdot\frac{\partial x_k}{\partial t_i}$$

Implicit Diff

If $F(x,y)=0$ defines y implicitly as a function of x.

Differentiate

$$\frac{dF}{dx} = \frac{\partial F}{\partial x}\cdot\frac{dx}{dx} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} = 0$$

Assume $\frac{\partial F}{\partial y} \neq 0$

$$\frac{\partial F}{\partial x} = -\frac{\partial F}{\partial y}\cdot\frac{dy}{dx} \implies$$

$$\frac{dy}{dx} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} \quad !$$

#s 27-30

$\frac{d}{dx}[xy] = y + xy'$

27 $\sqrt{xy} = 1 + x^2y$

Find $\frac{dy}{dx}$: $\frac{1}{2}(xy)^{-\frac{1}{2}}(y) + \frac{1}{2}(xy)^{-\frac{1}{2}}(xy') = 2xy + x^2y'$

Solve for y' etc.

Implicit Differentiation Theorem gives us a formula for doing this more efficiently.

$$\boxed{\frac{dy}{dx} = \frac{-F_x}{F_y}}$$

where F is found by getting everything on one side

$F(x,y) = \sqrt{xy} - 1 - x^2y = 0$