§ 15.3

§ 15.2 # 16

#17 $\lim_{\bar{x}\to\bar{0}} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

$x=0,\ y\to 0$: lim is 2

$y=0,\ x\to 0$: lim is 2

$y=x,\ x\to 0$: " " "

Conjecture $\lim_{\bar{x}\to\bar{0}} f(x,y) = 2$

WTS this $\to 0$

$$\left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} - 2\right| \xrightarrow{(x,y)\to(0,0)} 0$$

OR show

$$\left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\right| \xrightarrow{(x,y)\to(0,0)} 2$$

Here we go:

$$\left|\frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}\cdot\frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}\right|$$

$$= \left|\frac{(x^2+y^2)(\sqrt{x^2+y^2+1}+1)}{x^2+y^2+1-1^2}\right| = \left|\frac{\cancel{(x^2+y^2)}(\sqrt{x^2+y^2+1}+1)}{\cancel{x^2+y^2}}\right|$$

$$= \left|\sqrt{x^2+y^2+1}+1\right| \xrightarrow{(x,y)\to(0,0)} 2 \ \blacksquare$$

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$$f_x = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

(43) $f(x,y) = xy^2 - x^2y$

$$\frac{f(x+h,y) - f(x,y)}{h} = \frac{(x+h)y^2 - (x+h)^2y}{h}$$

$$= \frac{xy^2 + hy^2 - x^2y - 2xhy - h^2y}{h}$$

$$= \frac{-2xhy}{h} \xrightarrow{h \to 0} 2xy = f_x(x,y)$$

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46 $yz = \ln(x+z)$ Find $\frac{\partial z}{\partial x} = z_x = z'$

We assume z depends on x

But y doesn't.

$$\underbrace{y'z}_{0} + yz' = \frac{1+z'}{x+z}$$

Implicit Differentiation Probs.

$$yz'(x+z) = 1+z'$$

$$xyz' + yzz' - z' = 1$$

$$z'(xy + yz - 1) = 1$$

$$z' = \frac{1}{xy+yz-1}$$

Feb 24-12:26 PM

$2x - 6y + 8z = -16$ $\quad \bar{n}_1 = \langle 2, -6, 8 \rangle$

$2x - 5y + 7z = -14$ $\quad \bar{n}_2 = \langle 2, -5, 7 \rangle$

$$\begin{array}{r} \langle 2, -6, 8 \rangle \\ \times \quad \langle 2, -5, 7 \rangle \\ \hline \underline{\langle -2, 2, 2 \rangle} = \bar{d} \end{array}$$

is direction vector for the line they share.

$\underline{\langle -1, 1, 1 \rangle}$ is nicer.

$2x - 6y + 8z = -16$

$2x - 5y + 7z = -14$

Let $x = 0$ on the assumption that our line hits the yz-plane.

$-6y + 8z = -16$

$-5y + 7z = -14$

$$\begin{array}{r} 30y - 40y = +80 \\ -30y + 42y = -84 \\ \hline 2y = -4 \\ y = -2 \end{array}$$

$-6(-2) + 8z = -16$

$12 + 8z = -16$

$8z = -28$

$z = -\frac{14}{4} = -\frac{7}{2} = -\frac{7}{2}$

$x = 0, y = -2, z = -\frac{7}{2}$

Let $z = 0$:

$(-2, 2, 0)$ on $\mathcal{L}$

$x = x_0 + at = -2 - t$

$y = 2 + t$

$z = t$

$x = -2 - 2t$

$y = 2 + 2t$

$z = 2t$

$$
\begin{array}{r}
2x - 6y + 8z = -16 \\
-(2x - 5y + 7z = -14) \\
\hline
-y + z = -2
\end{array}
$$

$$2x - 4y + 8z = -16$$
$$-y + z = -2$$

$$-y = -z - 2$$
$$y = z + 2$$
$$2x - 6(z+2) + 8z = -16$$
$$2x - 6z - 12 + 8z = -16$$
$$2x + 2z - 12 = -16$$
$$2x = -2z - 4$$
$$x = -z - 2$$
$$z = z$$

$$\boxed{\begin{aligned} x &= -2 - t \\ y &= 2 + t \\ z &= t \end{aligned}}$$

§15.4 Tangent Plane

The last time in class, I discussed one way of thinking of the tangent plane. A little closer to the book's idea.

Take a surface, $f(x,y) = z$

what's f_x? Slope of the surface in the x-direction

What if I keep y fixed ($y=1$) and look at the line with slope f_x:

$$f(x,y) = 6 - x - x^2 - 2y^2$$

Let's look at it where $y=1$:

The trace of the plane $y=1$ is:

$$f(x,1) = 6 - x - x^2 - 2$$

$$z = -x^2 - x + 4$$

$$f_x = -1 - 2x$$

what's $\frac{dz}{dx}$, when we hold $y=1$?

$$\frac{dz}{dx} = -1 - 2x = f_x$$

Let $x=1$ be fixed.

Then $f(1,y) = 6-1-1^2-2y^2$

$= 4-2y^2$

$z = -2y^2+4$

what's the tangent to this curve at $y=1$?

$$\left.\frac{dz}{dy}\right|_{y=1} = \left.-4y\right|_{y=1} = -4$$

The tangent line in the y-direction is $z=-4(y-1)+2$

$z=-4(y-y_0)+z_0$

$z=f_y(y-y_0)+z_0$

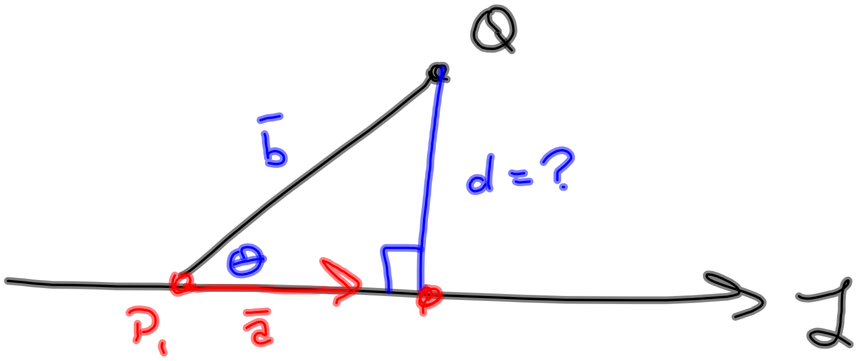
in x-direction

$z=f_x(x-x_0)+z_0$

$$z = f_x(x-x_0)+f_y(y-y_0)+z_0$$

$Q(5,4,77)$

$\mathcal{L}: \begin{matrix} 2+3t \\ 5-2t \\ 1+t \end{matrix}$ $\quad \bar{a} = \langle 3,-2,1 \rangle$, $P_1 = (2,5,1)$

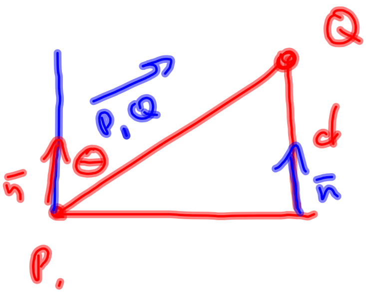


$$\frac{d}{|\overrightarrow{P_1Q}|} = \sin\theta = \frac{d}{|\bar{b}|} = \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|}$$

$\overrightarrow{PQ} = \bar{b} = \langle 3,1,76 \rangle$ $\qquad d = \dfrac{|\bar{a} \times \bar{b}|}{|\bar{a}|}$

$=$

Pt $\Rightarrow$ Plane



$$\boxed{\begin{aligned} \bar{B} &= \bar{T} \times \bar{N} \\ \bar{N} &= \frac{\bar{T}'}{|\bar{T}'|} \end{aligned}}$$