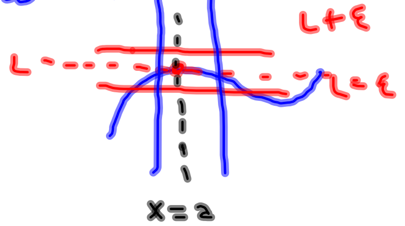
§15.2 Limits & Continuity

Recall $\lim_{x\to a} f(x) = L$ means,

Existence of Limit

Given any $\varepsilon > 0$, $\exists\ \delta > 0 \ni$

whenever $0 < |x-a| < \delta$, we have $|f(x) - L| < \varepsilon$



No escape from top or bottom of window if you're between

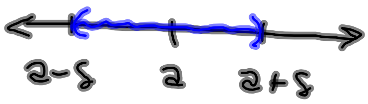
$a-\delta$ & $a+\delta$ ($x=a$ not included)

$$\frac{f(x+h)-f(x)}{h}$$

Continuity: f is cont^s at $x=a$ if $\lim_{x\to a} f(x) = f(a)$

Very similar for $f: \mathbb{R}^2 \longrightarrow \mathbb{R}$

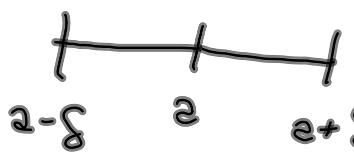
Instead of



A 1-ball

we now have

A 2-ball

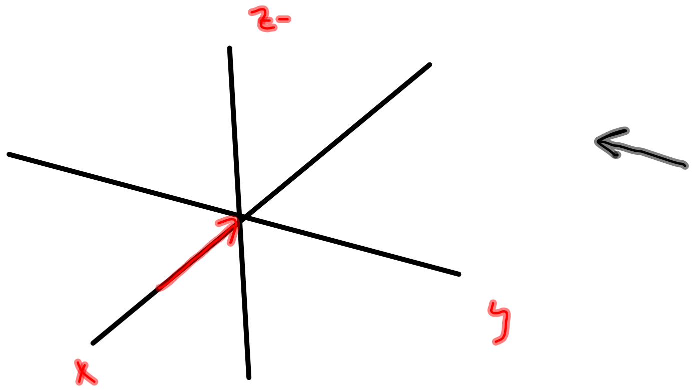


Surface (graph) of $f(x,y)$

The image of the open disc stays between the two planes

A CRAPPY continuity picture

$$\lim_{(x,y)\to(0,0)}$$

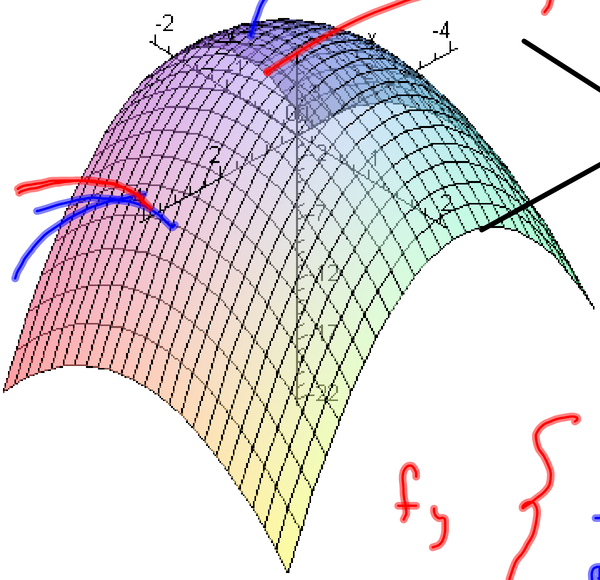


$$\frac{y^8 - y^4}{y^4 + y^2} \qquad \frac{y^4(y^4-1)}{y^4\left(1+\frac{1}{y^2}\right)} = \frac{y^4-1}{\frac{y^2+1}{y^2}}$$

$$= \frac{y^6-y^2}{y^2+1}$$

§15.3 Partial Derivatives.

$f_x = \frac{\partial f}{\partial x}$ $\quad f_y = \frac{\partial f}{\partial y}$



Taking derivatives w/ respect to one variable. Like taking the derivative of the intersection of a plane $x = c$ with the surface.

f_y $\left\{ \text{If } x = c \text{, then we're looking a cross-section parallel to the } yz\text{-plane.} \right.$

$$f(x,y) = y^5 - 3xy$$

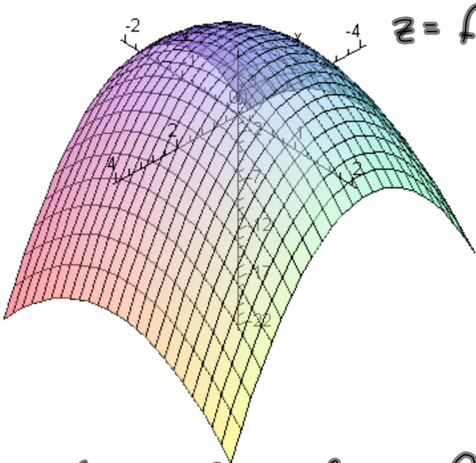
$$f_x = 3y$$

$$f_y = 5y^4 - 3x$$

Formally: $f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$

f_y is similarly defined.

How to find eq'n of the tangent plane?
Book has nice formula.
$z = f_x(x-x_0) + f_y(y-y_0) + z_0$



$z = 6 - x - x^2 - 2y^2 = f(x,y)$

$f_x = -1-2x$ @ $(1,2,-4)$

$f_y = -4y$

$f_x(1,2) = -1-2 = -3$

$f_y(1,2) = -8$

$z = f_x(x-x_0) + f_y(y-y_0) + z_0$

$$\boxed{z = -3(x-1) - 8(y-2) - 4}$$

Direction Vector for the line in the direction of f_x: $\langle 1, 0, -1-2x \rangle$

$f_x = -1-2x$

$\boxed{f_x(1,2) = -3}$

$f_y(1,2) = -8$

$\boxed{\langle 1, 0, -3 \rangle}$

$\langle 0, 1, -4y \rangle$

$\times \boxed{\langle 0, 1, -8 \rangle}$

$\langle 3, 8, 1 \rangle = \overline{n}$

My plane:

$3(x-1) + 8(y-2) + 1(z+4) = 0$

$z = -3(x-1) - 8(y-2) - 4$

$-3(x-1) - 8(y-2) - 4 - z = 0$

$3(x-1) + 8(y-2) + 1(z+4) = 0$

Feb 22-12:54 PM