http://www.trails.com/topomap.aspx?trailid=BGS114-069

Contours / Level Curves.

§ 15.1 Functions of 2 variables

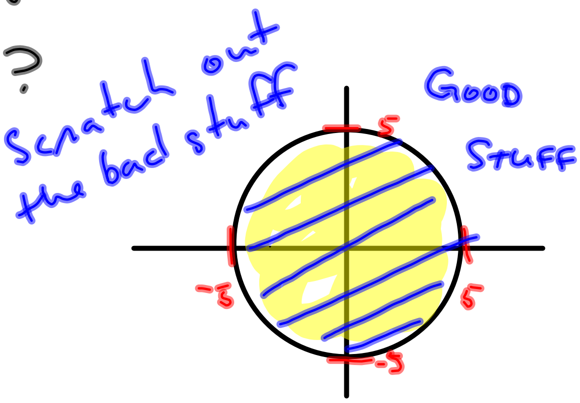
Surfaces embedded in 3-space

$z = f(x,y) = \sqrt{x^2+y^2-25}$

what's its domain?

$x^2+y^2-25 \geq 0$

$x^2+y^2 \geq 25$



$f(x,y) = \sqrt{x+3y-1} + \ln(2x+5y-7) + \sqrt{x}$

Sketch its domain

Need

$x+3y-1 \geq 0$ AND $2x+5y-7 > 0$

$x+3y=1$

x	y
0	$1/3$
1	0

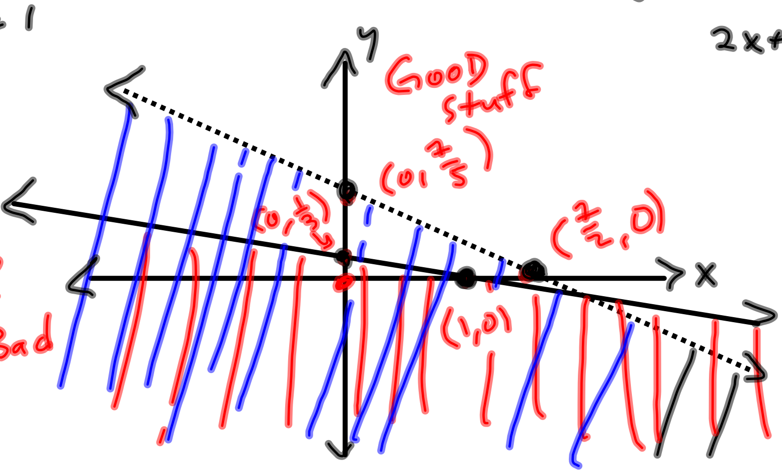
$2x+5y=7$

x	y
0	$7/5$
$\frac{7}{2}$	0

$x+y \geq 1$

$0 \geq 1$?

$(0,0)$ Bad



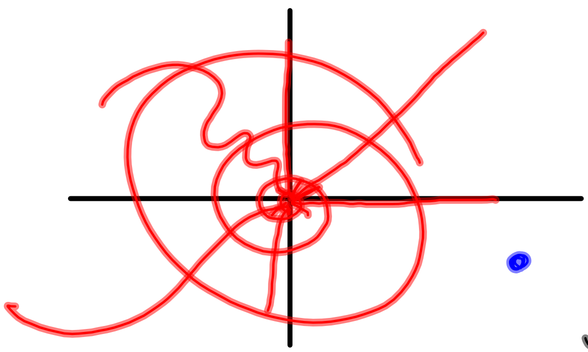
$0 \geq 7$?

$(0,0)$ Bad

§15.2 Limits & Continuity

$$\lim_{(x,y)\to(0,0)} \left(\frac{x^2+\sin^2 y}{2x^2+y^2} \right)$$

$(x,y) \to (0,0)$ can be along <u>any</u> path:



$x=0$:

$$\frac{x^2+\sin^2 y}{2x^2+y^2} = \frac{\sin^2 y}{y^2} = \left(\frac{\sin y}{y}\right)^2 \xrightarrow{y\to 0} 1$$

$y=0$:

$$\frac{x^2}{2x^2} = \frac{1}{2} \xrightarrow{x\to 0} \frac{1}{2}$$

Limit $\nexists$.

$\bar{x} = \langle x, y \rangle$ $\qquad f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}}$

$\bar{x} = \langle x_1, x_2 \rangle$ $\qquad f(x_1, x_2) = \dfrac{x_1 x_2}{\sqrt{x_1^2 + x_2^2}}$

$$\lim_{\bar{x} \to \bar{0}} f(x,y) = \lim_{\bar{x} \to 0} f(\bar{x})$$

$x = 0$: $\quad f(x,y) = \dfrac{0}{\sqrt{y^2}} = 0$

$y = 0$: $\quad f(x,y) = 0$

$y = x$: $\quad f(x,y) = \dfrac{x^2}{\sqrt{2x^2}} = \dfrac{x^2}{\sqrt{2}|x|}$ $\quad \to \dfrac{x^2}{\sqrt{2}x} = \dfrac{x}{\sqrt{2}}$ $\quad \to -\dfrac{x^2}{\sqrt{2}x} = -\dfrac{x}{\sqrt{2}}$

both pieces approach 0 as $\bar{x} \to \bar{0}$.

How do we prove the limit is zero.

Squeeze it, baby.

$f(x,y) = \dfrac{xy}{\sqrt{x^2+y^2}}$ $\qquad$ Look @ $|f(x,y)|$

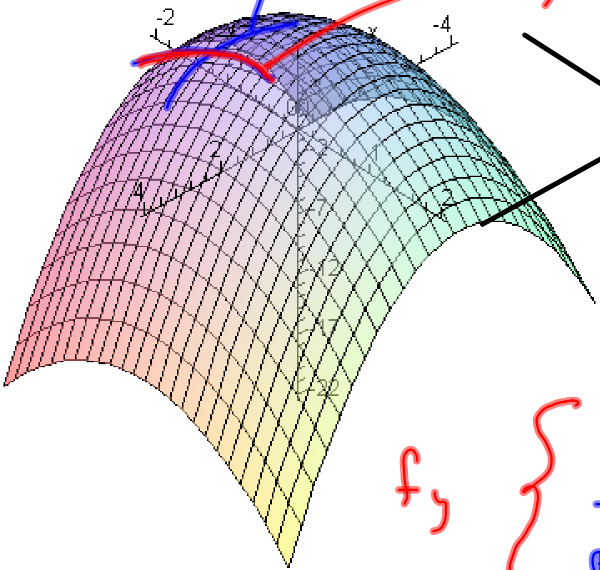
$$= \left| \frac{xy}{\sqrt{x^2+y^2}} \right|$$

Assume $|x| < 1$ & $|y| < 1$

$$\underset{\downarrow\, 0}{0} \le \underbrace{\left| \frac{xy}{\sqrt{x^2+y^2}} \right|}_{\downarrow\, 0} \le \left| \frac{xy}{\sqrt{x^2}} \right| \le \underset{\downarrow\, 0}{|y|} \xrightarrow{(x,y) \to (0,0)}$$

§15.3 Partial Derivatives.

$f_x = \frac{\partial f}{\partial x}$ $f_y = \frac{\partial f}{\partial y}$



Taking derivatives w/ respect to one variable. Like taking the derivative of the intersection of a plane $x = c$ with the surface.

f_y $\left\{ \text{If } x = c, \text{ then we're looking a cross-section parallel to the } yz\text{-plane.} \right.$

$$f(x,y) = y^5 - 3xy$$

$$f_x = 3y$$

$$f_y = 5y^4 - 3x$$

FORMALLY: $f_x = \frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$

f_y is similarly defined.

$$f_{xy} = \frac{\partial}{\partial y}\left(f_x\right) = \frac{\partial^2 f}{\partial y \partial x}$$

Mixed 2nd partials.

Clairaut said that, when things're runnin' smoothly,

$$f_{xy} = f_{yx}$$

$$f_{xx} = \frac{\partial^2 f}{\partial x^2}$$

§ 15.1 – 15.3 Due Friday