$$\kappa = \left| \frac{d\bar{T}}{ds} \right|$$

$$s(t) = \int_2^t |\bar{r}'(u)|\,du$$

$$\left| \frac{d\bar{T}}{ds} \right| = \left| \frac{\frac{d\bar{T}}{dt}}{\frac{ds}{dt}} \right| = \boxed{\frac{|\bar{T}'(t)|}{|\bar{r}'(t)|}}$$

$$\frac{|\bar{r}' \times \bar{r}''|}{|\bar{r}'|^3}$$

only practical way, with paper & pencil.

$\bar{T}'(t)$:

$$\bar{T} = \frac{\bar{r}'}{|\bar{r}'|} = \frac{1}{\sqrt{1+36t+4t^2}} \langle 1, 6t^{\frac{1}{2}}, -2t \rangle$$

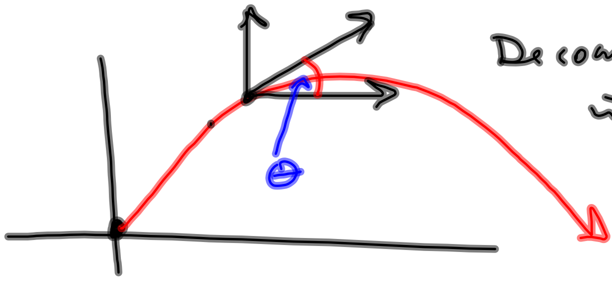
f g

$$\bar{T}' = f'g + fg' \neq fg'$$

§ 14.4 Solutions: Check 'em out.

This way of doing

$$\bar{a}(t) = a_{\bar{T}}\,\bar{T} + a_{\bar{N}}\,\bar{N}$$



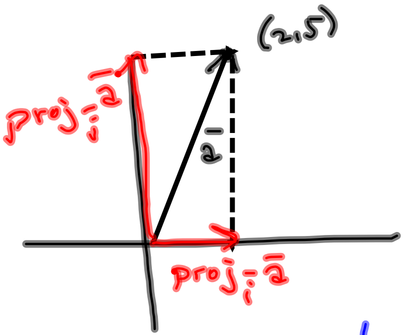
Decomposing a vector into its vertical and horizontal components.

$$\bar{r}(t) = \langle x(t), y(t) \rangle$$

$$= |\bar{r}(t)| \langle \cos\theta, \sin\theta \rangle$$

$$\left.\begin{array}{l}\bar{i} = \langle 1,0\rangle \\ \bar{j} = \langle 0,1\rangle\end{array}\right\}$$ Canonical (Standard) Basis for $\mathbb{R}^2$

$$\bar{a} = \langle 2,5\rangle = 2\langle 1,0\rangle + 5\langle 0,1\rangle$$



$|\bar{i}| = 1$
$|\bar{j}| = 1$

$$= \langle 2,0\rangle + \langle 0,5\rangle$$
$$= \text{proj}_{\bar{i}}\bar{a} + \text{proj}_{\bar{j}}\bar{a}$$
$$= (\text{comp}_{\bar{i}}\bar{a})\,\bar{i} + (\text{comp}_{\bar{j}}\bar{a})\,\bar{j}$$
$$= (\bar{a}\cdot\bar{i})\,\bar{i}$$

In the context of tangential & normal components, we have an orthonormal basis for $\mathbb{R}^2$: $\{\bar{T}, \bar{N}\}$, just like $\{\bar{i}, \bar{j}\}$ is.

$$\frac{|\text{comp}_{\bar{i}}\bar{a}|}{|\bar{a}|} = |\cos\theta|$$

$$\frac{|\bar{a}\cdot\bar{i}|}{|\bar{i}|} = |\bar{a}\cdot\bar{i}|$$

Now $\text{comp}_{\bar{i}}\bar{a}$ is signed

$$\underbrace{\text{ortho}}_{\perp}\ \underbrace{\text{normal}}_{|*|=1}$$

$\bar{a} = a_{\bar{T}}\bar{T} + a_{\bar{N}}\bar{N}$

$a_{\bar{T}} \to \text{comp}_{\bar{T}}\bar{a}$, $a_{\bar{N}} \to \text{comp}_{\bar{N}}\bar{a}$

$= (\bar{a}\cdot\bar{T})\bar{T} + (\bar{a}\cdot\bar{N})\bar{N}$

This is what I did on the 14.4 question, #33

I never found $a_{\bar{T}}$, $a_{\bar{N}}$

yeah ok, I did this one. Nevah!

13.3 #s 41, 42

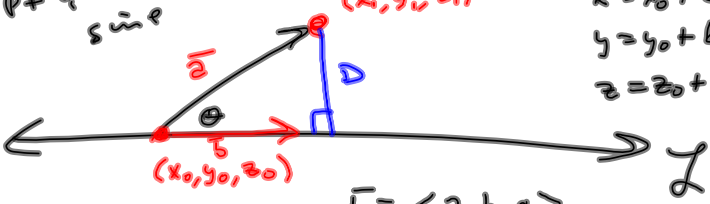
$\boxed{\bar{a} = (\bar{a}\cdot\bar{T})\bar{T} + (\bar{a}\cdot\bar{N})\bar{N}}$, then all I need to do is find $\bar{a}\cdot\bar{T}$ & $\bar{T}$!

$\bar{a} - (\bar{a}\cdot\bar{T})\bar{T} = \underbrace{(\bar{a}\cdot\bar{N})\bar{N}}$

GRAM-SCHMIDT ORTHOGONALIZATION?

Never have to calculate this! Just subtract!

pt & Line
sine



$x = x_0 + at$
$y = y_0 + bt$
$z = z_0 + ct$

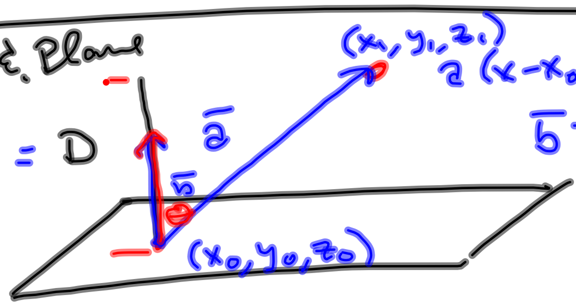
$\bar b = \langle a, b, c \rangle$

$$D = \sin\theta = \frac{|\bar a \times \bar b|}{|\bar b|}$$

$$|\bar a||\bar b|\sin\theta = |\bar a \times \bar b|$$

pt & Plane

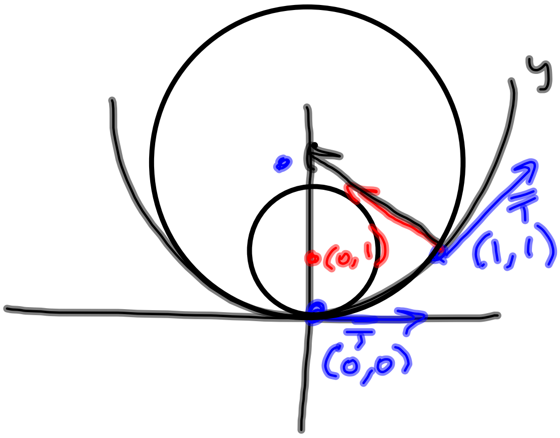
$|\mathrm{comp}_{\bar n}\bar a| = D$



$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$\bar b = \bar n = \langle a, b, c \rangle$

$$\frac{|\bar a \cdot \bar n|}{|\bar n|} = |\mathrm{comp}_{\bar n}\bar a|$$



$\bar{T}$ = slope = m = 1
$\bar{N} = m_\perp = -\frac{1}{1} = -1$

r=1, (0,0) is on it.

$\kappa(0) = 1 \Longrightarrow \rho = r = \frac{1}{\kappa}$

$\kappa(1) = \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{2\cdot 2} = \frac{\sqrt{2}}{4}$

$x^2 + (y-1)^2 = 1$

This other one:

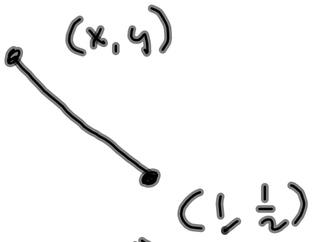
$r = \frac{1}{\kappa} = \frac{1}{\frac{1}{2\sqrt{2}}} = 2\sqrt{2}$

$m = \bar{T}(1) = \frac{d}{dx}\left(\frac{1}{2}x^2\right)\Big|_{x=1} = x\Big|_{x=1} = 1 = m$

A line of $m_\perp = -1$ containing $(1, \frac{1}{2})$

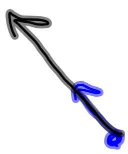
Need to go $2\sqrt{2}$ in the direction of $m_\perp = -1$ from $(1, \frac{1}{2})$

unit vector in the direction of $y=-x$



$$\frac{\langle -1, 1\rangle}{\sqrt{1^2+1^2}} = \frac{\langle -1, -1\rangle}{\sqrt{2}}$$

Now go $2\sqrt{2}$ units from $\langle 1, \frac{1}{2}\rangle$ in that direction:



$$\langle 1, \tfrac{1}{2}\rangle + 2\sqrt{2}\,\frac{\langle -1, 1\rangle}{\sqrt{2}} =$$

$$= \langle 1, \tfrac{1}{2}\rangle + \langle -2, 2\rangle$$

$$= \langle -1, \tfrac{5}{2}\rangle$$

$$(x+1)^2 + (y-\tfrac{5}{2})^2 = 8$$