$$\kappa = \frac{|\bar{T}'(t)|}{|\bar{r}'(t)|} = \frac{|\bar{r}'(t) \times \bar{r}''(t)|}{|\bar{r}'(t)|^3}$$

$\bar{r}$ given

Better when $|\bar{r}'(t)| = \text{constant}.$

Recognize when the curve lives in one of our basic 3-D shapes.

$\cos^2 t + \sin^2 t = 1$

$\langle \cos t, \sin t, t \rangle$

$\langle t^2 \sin(2t), t, 73t^2 - 5 \rangle$

$\langle \ln(t), \cos(7t), 25t^3 \rangle$

$\bar{r}(t) = \langle \cos t, t, \sin t \rangle$

$\bar{r}'(t) = \langle -\sin t, 1, \cos t \rangle$

$\Rightarrow |\bar{r}'(t)| = \sqrt{2}$

Line shared by 2 planes

$ax+by+cz=d$

$ex+fy+gz=h$

$\mathcal{S}$ $\bar{u}\cdot(\bar{v}\times\bar{w})=2.$

what's...

$(\bar{u}\times\bar{v})\cdot\bar{w}=2$ T 13.4.8

Let $A(1,0,1)$, $B(2,3,0)$, $C(-1,1,4)$, $D(0,3,2)$

Plane containin' A, B, & C

Area of ΔABC

Area of parallelogram defined by $\overrightarrow{AB}$ & $\overrightarrow{AD}$

Volume of parallelepiped with adjacent edges $\overrightarrow{AB}$, $\overrightarrow{AC}$, $\overrightarrow{AD}$

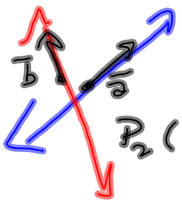
Distance from D to plane containing A, B, C

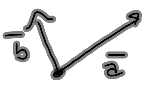
Distance between

$P_1(2,1,0)$ $\mathcal{L}_1$: $x=2+t$, $y=1+3t$, $z=t$ $\bar{a}=\langle 1,3,1\rangle$

$\mathcal{L}_2$: $x=5$, $y=8-8t$, $z=3-4t$ $\bar{b}=\langle 0,-8,-4\rangle$

$\bar{a}\times\bar{b}=\bar{n}$ to both $\bar{a}\times\bar{b}=\bar{c}$





$\bar{a}\times\bar{b}$ is $\perp$ to $\mathcal{L}_1$ & $\mathcal{L}_2$?

2,1,0) $P_2(5,8,3)$

Does the plane

$$\left|-1(x-2)+(y-1)-2(z-0)\right|=0$$

$\sqrt{1^2+1^2+2^2}$

contain $\mathcal{L}_1$?

$$\begin{array}{r} \langle 1,\ 3,\ 1\rangle \\ \times\ \langle 0,-8,-4\rangle \\ \hline \langle -4,4,-8\rangle \end{array}$$

OR $\langle -1,1,-2\rangle$ yeah.

$-1(x-5)+(y-8)-2(z-3)=0$

contain $\mathcal{L}_2$?

Double-check it 2B sure.

a find distance from ~~the~~ plane to a point on the other plane.

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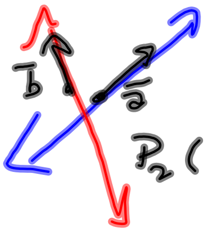
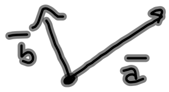




$\bar{a} \times \bar{b}$ is $\perp$ to $\mathcal{L}_1$ & $\mathcal{L}_2$?

$P_2(5,8,3)$

2,1,0)

$$\begin{array}{r} \langle 1,\ 3,\ 1\rangle \\ \times\ \langle 0, -8, -4\rangle \\ \hline \langle -4, 4, -8\rangle \end{array}$$

OR $\langle -1, 1, -2\rangle$ yeah.

Double-check it 2B sure.

a

Does the plane

$$\left| -1(x-2)+(y-1)-2(z-0) \right| = 0$$

$\sqrt{1^2+1^2+2^2}$

contain $\mathcal{L}_1$?

$$-1(x-5)+(y-8)-2(z-3)=0$$

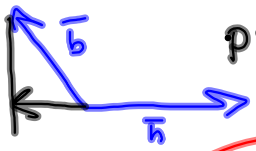
contain $\mathcal{L}_2$?

Find distance from ~~the~~ plane to a point on the other plane.

$\bar{a} \bullet \bar{b}$

$\bar{a} \times \bar{b}$

$\text{proj}_{\bar{n}} \bar{b} = \bar{c}$



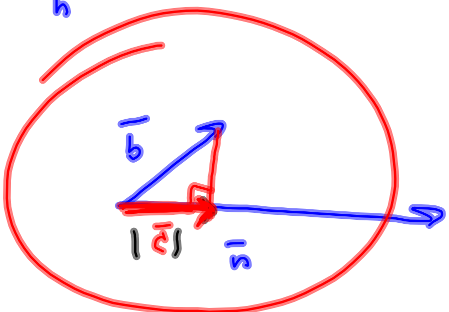
$= |\bar{c}|$

$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta$

$\bar{a} \cdot \bar{b} = |\bar{a}||\bar{b}| \cos\theta$

$\boxed{|\bar{c}| = |\bar{b}||\cos\theta|}$

$\frac{|\bar{c}|}{|\bar{b}|} = |\cos\theta| = \frac{|\bar{b} \cdot \bar{n}|}{|\bar{b}||\bar{n}|}$



Want $|\bar{c}|$ times a unit vector in the direction of $\bar{n}$

unit vector times that length

$|\bar{c}| = |\bar{b}| \cos\theta = \frac{|\bar{b} \cdot \bar{n}|}{|\bar{n}|}$

Now, vector with $|\bar{c}|$'s length in the proper direction:

$$\text{Proj}_{\bar{n}} \bar{b} = \frac{|\bar{b} \cdot \bar{n}|}{|\bar{n}|} \cdot \frac{\bar{n}}{|\bar{n}|} = \frac{|\bar{b} \cdot \bar{n}|}{|\bar{n}|^2} \bar{n}$$

$\bar{r}(t) = \langle t, 2\cos t, \sin t \rangle$

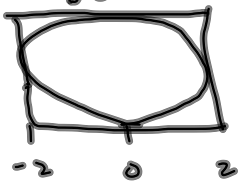
$\cos^2 t + \sin^2 t = 1$

$\frac{2^2\cos^2 t}{2^2} = \cos^2 t$

$\frac{x^2}{2^2} + y^2 = 1$

yz-plane projection

$\frac{2\cos t}{2} = \cos t$



$m\frac{dv}{dt} = \frac{dm}{dt} v_e$

Find $v(t)$

v_e = constant = escape velocity.

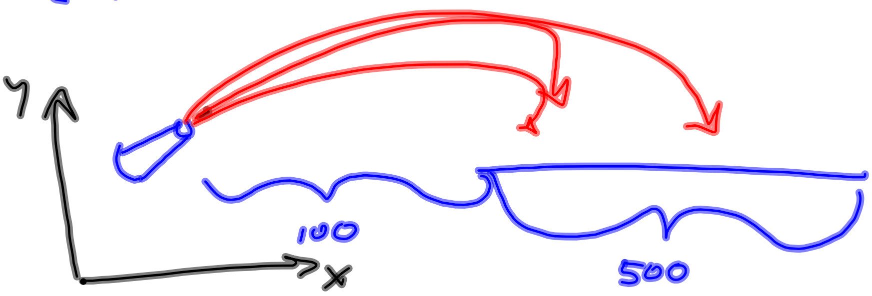
$m = m(t)$

$\frac{dv}{dt} = \frac{\frac{dm}{dt}}{m(t)} v_e$

$\int \frac{dv}{dt} dt = \int \frac{\frac{dm}{dt}}{m(t)} v_e \, dt$

$\int dv = v_e \int \frac{dm}{m}$

Muzzle velocity is $80 \frac{m}{s}$
what angles of elevation will drop our canisters inside the fort?



Fort w/ no walls.

Want $100 \le x(t) \le 600$
when $y(t) = 0$ → FOR ACTUAL #29, need $y(t) = 15\,m$

$v_0 = 80\,m/s$

$\vec{r}(t) = \langle (80\cos\theta)t, (80\sin\theta)t - 4.9t^2 \rangle$

when is $y = 0$?

$$80t\sin\theta - 4.9t^2 = 0$$

$$t(4.9t - 80\sin\theta) = 0$$

$t = 0$ or $4.9t = 80\sin\theta$

$$t = \frac{80\sin\theta}{4.9}$$

Set $x = (80\cos\theta)\left(\frac{80\sin\theta}{4.9}\right) = 100$

$$\frac{6400}{4.9}\sin\theta\cos\theta = 100$$

$$2\sin\theta\cos\theta = \frac{2(100)(4.9)}{6400}$$

$$\sin(2\theta) = \frac{(100)(4.9)}{3200}$$

$$2\theta = \arcsin\left(\frac{(100)(4.9)}{3200}\right)$$

$$2\theta \approx 8.808068561^\circ$$

$$\theta \approx 4.40403428^\circ$$

Range of inverse sine is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$180^\circ - 8.808^\circ$ also satisfies

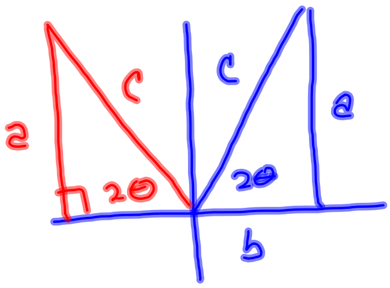
$$\sin(2\theta) = \frac{(100)(4.9)}{3200}$$

$$171.192^\circ \approx 2\theta$$

$$\theta \approx \frac{171.192}{2} \approx 85.596^\circ$$

↑ is another solution.

Two trajectories that reach 100 m, exactly.



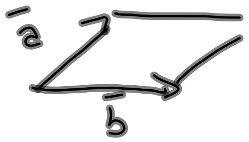
Both have same, positive sine.

So make sure you're not missing a candidate angle.

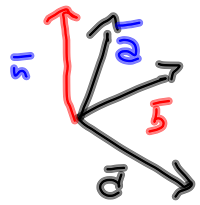
This happens in #23 or #27

$$|\bar{a}\times\bar{b}| = |\bar{a}||\bar{b}|\sin\theta$$

$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$$



$|\bar{a}\times\bar{b}|$



$$|\bar{a}\cdot(\bar{b}\times\bar{c})|$$

$\bar{r}(t) = \langle \cos t, \sin t, \ln(\cos(t))\rangle$

Find $\bar{T}, \bar{N}, \bar{B}$ @ $(1,0,0)$

$\bar{r}'(t) = \langle -\sin(t), \cos(t), -\tan(t)\rangle$

$$\bar{T}(t) = \frac{\bar{r}'}{|\bar{r}'|}$$

$$|\bar{r}'(t)| = \sqrt{\sin^2 t + \cos^2 t + \tan^2 t} = \sqrt{1+\tan^2 t} = \sec t$$

etc.

where does $\bar{r} = \langle t, 0, 2t - t^2\rangle$ intersect $z = x^2 + y^2$?

$\bar{r}(t) = \langle \sin(3t), -\cos(3t), t\rangle$

Find $\displaystyle\int_0^{\frac{\pi}{2}} \bar{r}(s)\,ds$ $\qquad$ $\displaystyle\int_0^{\frac{\pi}{2}} \bar{r}(s)\,dt$

$$= \int_0^{\frac{\pi}{2}} \bar{r}(w)\,dw \qquad = t\,\bar{r}(s)\Big]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}\bar{r}(s)$$

$\boxed{\text{Test Monday}}$

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