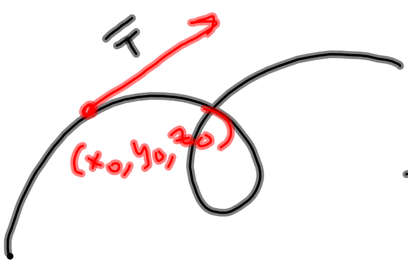
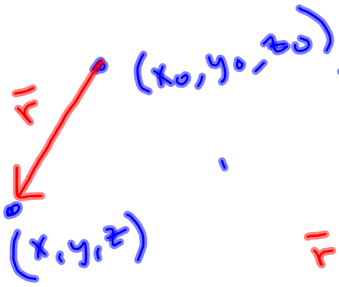


Normal Plane.

$\bar{T} = \langle a, b, c \rangle$ = normal to plane $= \bar{n}$



$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

If (x,y,z) is a point in the plane, then

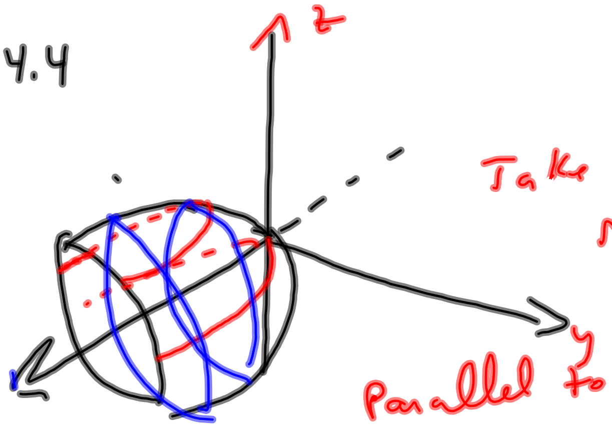
$\bar{r} = \langle x-x_0, y-y_0, z-z_0 \rangle$ is parallel to the plane.

So $\bar{n} \cdot \bar{r} = 0$

$ax + by + cz + d = 0$
meh

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$
Yeah.

§14.4



Take more slices.
Not just
$x=0, y=0,$ or $z=0$

Parallel to yz-plane.

§14.3 #s 43, 44 → Maple/Grapher Hold off, maybe

49, 53 → Projects? Hold off

$$\int \bar{v}'(t)\,dt = \bar{v}(t) + \bar{c}$$

§14.4 $\bar{r}$ = position vector = $\bar{r}(t)$

$\bar{v}(t)$ = velocity vector = $\bar{r}'(t)$

$\bar{a}(t)$ = acceleration = $\bar{v}'(t) = \bar{r}''(t)$

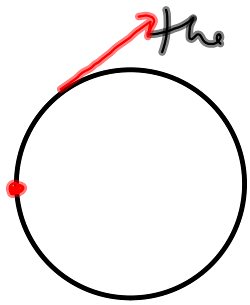
$$|\bar{v}(t)| = |\bar{r}'(t)| = \frac{ds}{dt} = \text{speed.}$$

Rocket Science: $F = ma$

$\bar{F} = m\bar{a}$

$\bar{F}(t) = m(t)\,\bar{a}(t)$

Example 4: Moving around in a circle, the force is always directed inward. The particle wants to move in a straight line.



$g = |\bar{a}| = 9.8\ m/s^2$ from gravity.

$\bar{r} = \langle x, y \rangle$

$\bar{r}(t) = \langle x(t), y(t) \rangle = x(t)\,\bar{i} + y(t)\,\bar{j}$

$\bar{a} = -g\,\bar{j}$

A projectile is shot with an initial velocity $\bar{v}_0 = \langle 50, 50 \rangle$

from a position $\bar{r}_0 = \langle 20, 10 \rangle$

Where is it, when $t = 3$ seconds?

$\bar{a}(t) = \langle 0, -9.8 \rangle$

$\Rightarrow \bar{v}(t) = \int \bar{a}(t)\,dt + \bar{C}$

$\frac{50}{9.8}$

$= \int \langle 0, -9.8 \rangle\,dt + \bar{C}$

$= \langle 0, -9.8t \rangle + \langle C_1, C_2 \rangle$

and we know that when $\underbrace{t=0,\ \bar{v}(t) = \langle 50, 50 \rangle}$

$\bar{v}(0) = \langle 50, 50 \rangle = \bar{v}_0$

That means

$\bar{v}(0) = \langle 0, -9.8(0)\rangle + \langle c_1, c_2\rangle = \langle 50, 50\rangle$

$\Longrightarrow \quad c_1 = c_2 = 50 \quad \&$

$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$

$v(t) = \langle 50, -9.8t + 50\rangle$

$$\bar{r}(t) = \int \bar{v}(t)\,dt + \bar{D}$$

$$= \int \langle 50, -9.8t + 50\rangle dt + \langle d_1, d_2\rangle$$

$$= \langle 50t, -\frac{9.8t^2}{2} + 50t\rangle + \langle d_1, d_2\rangle$$

and $\bar{r}(0) = \bar{r}_0 = \langle 20, 10\rangle = \langle d_1, d_2\rangle$

So $\bar{r}(t) = \langle 50t, -4.9t^2 + 50t\rangle + \langle 20, 10\rangle$

$$= \langle 50t + 20, -4.9t^2 + 50t + 10\rangle$$

$$= (50t + 20)\bar{i} + (-4.9t^2 + 50t + 10)\bar{j}$$

$v = |\bar{v}| = \text{speed}.$

$\bar{T} = \dfrac{\bar{r}'}{|\bar{r}'|} = \dfrac{\bar{v}}{|\bar{v}|} = \dfrac{\bar{v}}{v} = \dfrac{1}{v}\bar{v}$

κ

K $\quad v\bar{T} = \bar{v}$

$\boxed{5}$ $\quad \bar{a} = \bar{v}' = (v\bar{T})' = v'\bar{T} + v\bar{T}'$

$\boxed{6}$ $\quad K = \dfrac{|\bar{T}'|}{|\bar{v}|} = \dfrac{|\bar{T}'|}{|\bar{r}'|} = \dfrac{|\bar{T}'|}{v} \Longrightarrow$

$|\bar{T}'| = \kappa v$

$\bar{T}' = |\bar{T}'|\bar{N} = \kappa v \bar{N}$

$\therefore \quad \bar{a} = v'\bar{T} + \underbrace{\kappa v^2 \bar{N}}$

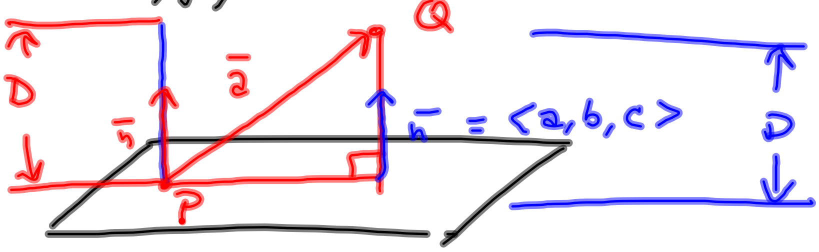
2/13 START

Distance $\sqrt{(x_1-x_0)^2+(y_1-y_0)^2+(z_1-z_0)^2}$

Pt. & a plane.

Q (x_1,y_1,z_1) $\quad ax+by+cz=d$ $\quad \bar{a}=\overrightarrow{PQ}$

P(x_0,y_0,z_0) on the plane. $\quad =\langle x_1-x_0, y_1-y_0, z_1-z_0\rangle$



$\text{proj}_{\bar{n}}\bar{a}=\frac{\bar{n}\cdot\bar{a}}{|\bar{n}|^2}\bar{n}$

$$D=|\text{comp}_{\bar{n}}\bar{a}|=\frac{|\bar{n}\cdot\bar{a}|}{|\bar{n}|}=\frac{|a(x_1-x_0)+b(y_1-y_0)+c(z_1-z_0)|}{\sqrt{a^2+b^2+c^2}}$$

Distance from Q to $\mathcal{L}$

$\mathcal{L}$: $x=x_0+at$ $\quad \bar{b}=\langle a,b,c\rangle$ is direction of $\mathcal{L}$.

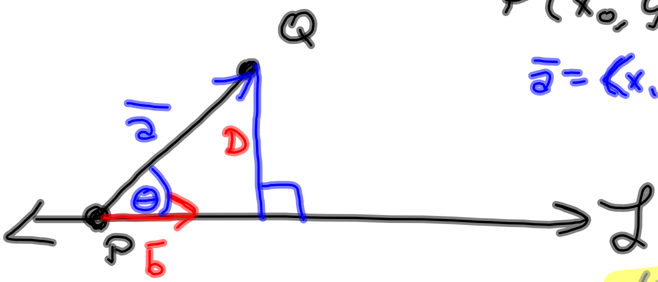
$y=y_0+bt$

$z=z_0+ct$

Q(x_1,y_1,z_1)

P(x_0,y_0,z_0)

$\bar{a}=\langle x_1-x_0, y_1-y_0, z_1-z_0\rangle$



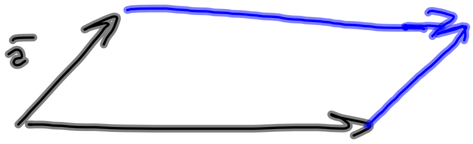
$|\bar{a}\times\bar{b}|=|\bar{a}||\bar{b}|\sin\theta$

$\sin\theta=\frac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}$

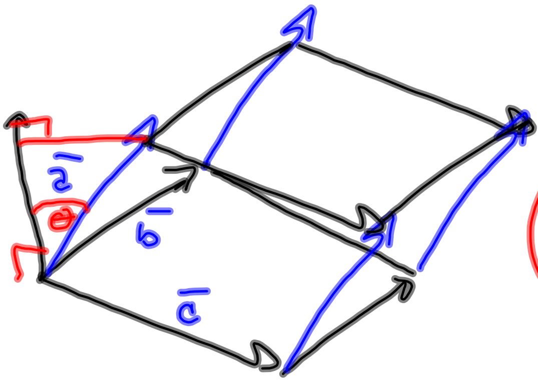
$$\frac{D}{|\bar{a}|}=\sin\theta=\frac{|\bar{a}\times\bar{b}|}{|\bar{a}||\bar{b}|}$$

$$\Rightarrow \boxed{D=\frac{|\bar{a}\times\bar{b}|}{|\bar{b}|}}$$

$\bar{a}\cdot\bar{b}=|\bar{a}||\bar{b}|\cos\theta$



area $= |\bar a \times \bar b|$



Volume

$$|\bar a \cdot (\bar b \times \bar c)|$$

h is $\perp$ to $\bar b$ & $\bar c$, i.e., in the direction of $\bar b \times \bar c$

$$\frac{|\bar h|}{|\bar a|} = \cos\theta = \frac{\bar a \cdot \bar h}{|\bar a||\bar h|} \Longrightarrow |\bar h| = \frac{|\bar a \cdot (\bar b \times \bar c)|}{\cancel{|\bar a|}|\bar h|}\cancel{|\bar a|}$$

$=$

Want $|\bar h|$ times area of the base.

$$\underbrace{|\bar h| \cdot |\bar b \times \bar c|}_{\text{volume}} = \frac{|\bar a \cdot (\bar b \times \bar c)|}{\cancel{|\bar h|}} \cdot \cancel{|\bar b \times \bar c|} = |\bar a \cdot (\bar b \times \bar c)|$$

$\to |\bar b \times \bar c|$

$< t, 0, \cos t >$

$= t\bar{i} + \cos t\,\bar{k}$

$|\bar{a} \cdot (\bar{b} \times \bar{c})|$ is calculated how?

$$\left| \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \right| = \left| a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} \right| =$$

$$|a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1)|$$

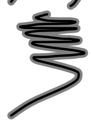
0

§ 14.3 #45

$$\bar{r}(t) = \langle 2\sin(3t), t, 2\cos(3t) \rangle$$

Normal plane

Osculating plane $\bar{T} \times \bar{N}$



$\bar{N} = \dfrac{\bar{T}'(t)}{|\bar{T}'(t)|}$

$$\bar{r}'(t) = \langle 6\cos(3t), 1, -6\sin(3t) \rangle$$

@ $(0, \pi, -2)$

what's t @ $(0, \overset{t}{\pi}, -2)$?

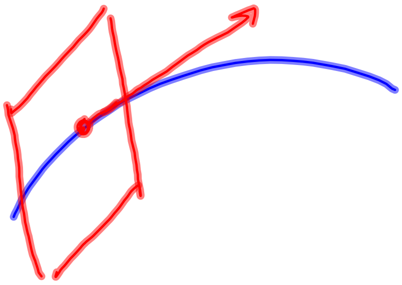
$$\bar{r}'(\pi) = \langle -6, 1, 0 \rangle = \bar{n} = \langle a, b, c \rangle$$

$$-6(x-0) + 1(y-\pi) + 0(z+2) = 0$$

$$-6x + y - \pi = 0$$

$$-6x + y = \pi$$

$\bar{r}(t) = <$



$$\bar{T}(t)\Big|_{t=\pi} = \frac{\bar{r}'(t)}{|\bar{r}'(t)|}\Big|_{t=\pi} = \frac{1}{\sqrt{37}} \langle -6, 1, 0 \rangle$$

$$\bar{r}(t) = \langle 2\sin(3t), t, 2\cos(3t) \rangle$$

$$\bar{r}'(t) = \langle 6\cos(3t), 1, -6\sin(3t) \rangle$$

a $(0, \pi, -2)$

$$\bar{r}'(\pi) = \langle 6\cos(3\pi), 1, -6\sin(3\pi) \rangle$$

$$= \langle -6, 1, 0 \rangle, \text{ so}$$

$$|\bar{r}'(\pi)| = \sqrt{6^2 + 1^2} = \sqrt{37}$$

$$\bar{T}(t) = \frac{1}{\sqrt{37}} \langle 6\cos(3t), 1, -6\sin(3t) \rangle$$

$$\bar{T}'(t) = \langle -18\sin(3t), 0, -18\cos(3t) \rangle$$

$$|\bar{T}'(t)| = 18 \implies$$

$$\bar{N}(t) = \frac{1}{18} \langle -18\sin(3t), 0, -18\cos(3t) \rangle$$

$$= \langle -\sin(3t), 0, -\cos(3t) \rangle$$

$$\bar{T}(t) \times \bar{N}(t) = \frac{1}{\sqrt{37}} \begin{vmatrix} i & j & k \\ 6\cos(3t) & 1 & -6\sin(3t) \\ -\sin(3t) & 0 & -\cos(3t) \end{vmatrix}$$

$$= \frac{1}{\sqrt{37}} \left[-\cos(3t)\,\bar{i} - \left(-6\cos^2(3t) - 6\sin^2(3t)\right)\bar{j} + (\sin(3t))\,\bar{k} \right]$$

$$= \frac{1}{\sqrt{37}} \langle -\cos(3t), 6, \sin(3t) \rangle = \bar{B}$$

$(0, \pi, -2)$: $t = \pi$

$$\bar{B}(\pi) = \frac{1}{\sqrt{37}} \langle 1, 6, 0 \rangle$$

$$\Rightarrow \frac{1}{\sqrt{37}}(x - 0) + \frac{6}{\sqrt{37}}(y - \pi) + 0(z+2) = 0$$

2

$$\frac{1}{\sqrt{37}} x + \frac{6}{\sqrt{37}}(y - \pi) = 0$$

Osculating Plane

$$\boxed{x + 6y = 6\pi}$$

Normal Plane: $-6x + y = \pi$

They're orthogonal:

$\bar{n}_1 = \langle 1, 6, 0 \rangle$

$\bar{n}_2 = \langle -6, 1, 0 \rangle$

$\bar{n}_1 \cdot \bar{n}_2 = 0$ ✓