Find eq'n of 2 planes ∥ to $x+2y-3z=1$ with distance 2 from it.

$\bar{n} = \langle 1,2,-3\rangle$

$x=0=y \implies -3z=1 \implies z=-\frac{1}{3}$

$(0,0,-\frac{1}{3})$ is on the plane.

2 units from there in the direction of $\bar{n}=\langle 1,2,-3\rangle$ will give us a pt. on the new plane.

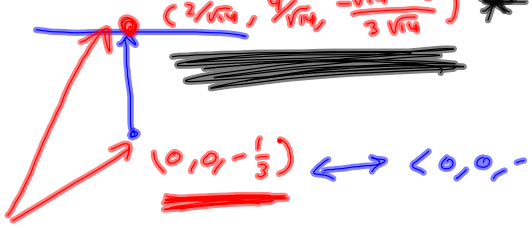
$$\frac{\bar{n}}{|\bar{n}|} = \frac{\langle 1,2,-3\rangle}{\sqrt{1^2+2^2+3^2}} = \frac{1}{\sqrt{14}}\langle 1,2,-3\rangle$$

So, $\frac{2}{\sqrt{14}}\langle 1,2,-3\rangle$ is a vector in $\bar{n}$'s direction that's 2 units.

$$-\frac{1}{3}+\frac{2}{\sqrt{14}}(-3) = -\frac{\sqrt{14}-6\cdot 3}{3\sqrt{14}} = \frac{-\sqrt{14}-18}{3\sqrt{14}}$$

so $\langle 0,0,-\frac{1}{3}\rangle + \frac{2}{\sqrt{14}}\langle 1,2,-3\rangle =$

$$\left\langle \frac{2}{\sqrt{14}}, \frac{4}{\sqrt{14}}, \frac{-\sqrt{14}-18}{3\sqrt{14}} \right\rangle$$



$\bar{n} = \langle 1,2,-3\rangle$

$$1\left(x-\frac{2}{\sqrt{14}}\right)+2\left(y-\frac{4}{\sqrt{14}}\right)-3\left(z-\frac{-\sqrt{14}-18}{3\sqrt{14}}\right)=0$$

For another plane 2 units away, use $-\bar{n}$

$|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$|2\bar{a}| = \sqrt{(2a_1)^2 + (2a_2)^2 + (2a_3)^2} = \sqrt{2^2(a_1^2 + a_2^2 + a_3^2)} = 2\sqrt{a_1^2 + a_2^2 + a_3^2} = 2|\bar{a}|$

find eq'n of plane

$x + 2y - 3z = 1$

Let $y = z = 0 \Rightarrow \langle 1, 0, 0 \rangle$ is on the plane.

$\frac{\bar{n}}{|\bar{n}|} = \frac{1}{\sqrt{14}}\langle 1, 2, -3 \rangle$

So, $\frac{2}{\sqrt{14}}\langle 1, 2, -3 \rangle$ is a vector in $\bar{n}$'s direction of length 2.

So we move 2 units away from $\langle 1, 0, 0 \rangle$ in that direction:

$$\langle 1, 0, 0 \rangle + \frac{2}{\sqrt{14}}\langle 1, 2, -3 \rangle = \left\langle \frac{\sqrt{14}+2}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}} \right\rangle$$

$$\longleftrightarrow \left(\frac{\sqrt{14}+2}{\sqrt{14}}, \frac{2}{\sqrt{14}}, -\frac{3}{\sqrt{14}} \right)$$

$3\langle 1, 2, 3 \rangle = \langle 3, 6, 9 \rangle$

$3(1, 2, 3)$ isn't defined!

§14.2

$\frac{d}{dt}$, $\int dt$ for vector-valued functions

Recall $(fg)' = f'g + fg'$

$\bar{r}(t) = \langle x(t), y(t), z(t) \rangle \longrightarrow$

$\frac{d}{dt}[\bar{r}(t)] = \bar{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$

$\int \bar{r}(t)\,dt = \langle \int x(t)\,dt, \int y(t)\,dt, \int z(t)\,dt \rangle$

Component-wise differentiation and integration.

$\frac{d}{dt}[\bar{r}(t) \bullet \bar{s}(t)] = \bar{r}'(t) \bullet \bar{s}(t) + \bar{r}(t) \bullet \bar{s}'(t)$

$(f \cdot g)' = f' \cdot g + f \cdot g'$

$\frac{d}{dt}[\bar{f}(t) \times \bar{g}(t)] = \bar{f}'(t) \times \bar{g}(t) + \underbrace{\bar{f}(t) \times \bar{g}'(t)}_{= -\bar{g}'(t) \times \bar{f}(t)}$

$\frac{d}{dt}[\bar{r} \times \bar{r}'] = ? = \bar{r} \times \bar{r}''$

$\bar{r}' \times \bar{r}' + \bar{r} \times \bar{r}''$!?

$$\bar{T}(t) = \frac{\bar{r}'(t)}{|\bar{r}'(t)|}$$

unit tangent vector.