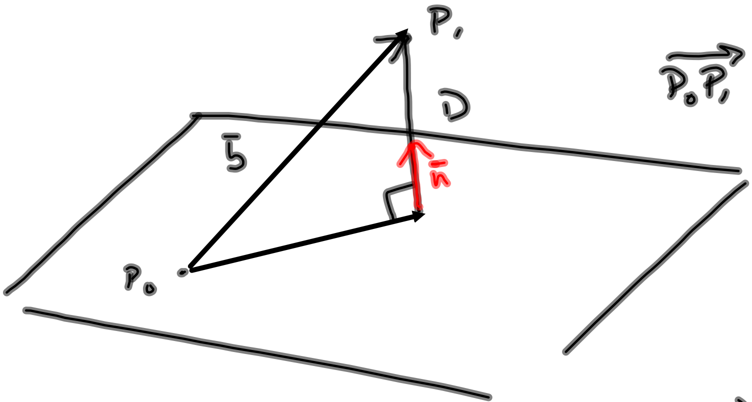
Distance from pt. to plane.



$$\overrightarrow{P_0P_1} = \langle x_1-x_0, y_1-y_0, z_1-z_0 \rangle = \bar{b}$$

$\mathcal{P}:\ a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$P_0(x_0,y_0,z_0)$ $\quad P_1(x_1,y_1,z_1)$

$\bar{n} = \langle a,b,c \rangle$

$$D = |\mathrm{comp}_{\bar{n}}\,\bar{b}| = \frac{|\bar{b}\cdot\bar{n}|}{|\bar{n}|}$$

$$= \frac{|\bar{n}\cdot\bar{b}|}{|\bar{n}|} = \frac{|a(x_1-x_0)+b(y_1-y_0)+c(z-z_0)|}{\sqrt{a^2+b^2+c^2}} \checkmark$$

Book says: $ax+by+cz+d=0$ is $\mathcal{P}$

My eq'n: $\mathcal{P}:\ a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$$\text{So } D = \frac{|ax_1+by_1+cz_1-(\underbrace{\boxed{ax_0+by_0+cz_0}}_{=-d})|}{\sqrt{a^2+b^2+c^2}}$$

$$= \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$ Book. Bleah.

Distance from pt to line

$\mathcal{L}$: $x = x_0 + at$, $y = y_0 + bt$, $z = z_0 + ct$

$P_0(x_0, y_0, z_0) \in \mathcal{L}$

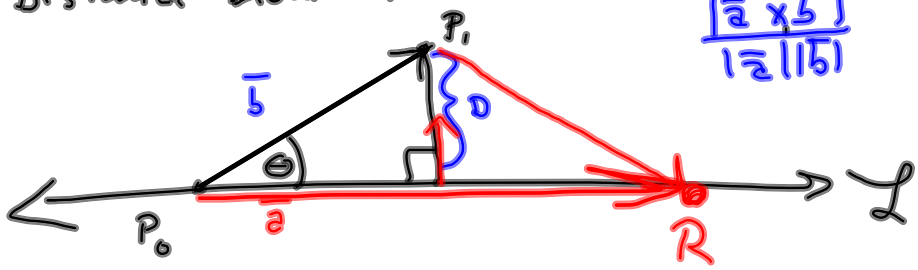
Direction vector for $\mathcal{L}$: $\langle a, b, c \rangle = \bar{a}$

$P_1(x_1, y_1, z_1) \notin \mathcal{L}$

Distance from P_1 to $\mathcal{L}$

$$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta$$

$$\frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|} = \sin\theta$$



TRIG

$\bar{b} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

Want D :

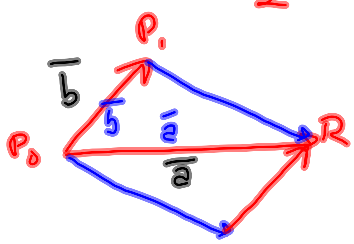
$$\frac{D}{|\bar{b}|} = \sin\theta \Longrightarrow D = |\bar{b}| \sin\theta$$

$$= |\bar{b}| \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|} = \boxed{\frac{|\bar{a} \times \bar{b}|}{|\bar{a}|} = D}$$

AREA

Area of the ΔP_0P_1R is half the area of the parallelogram

$$\frac{1}{2}|\bar{a}|D = \frac{1}{2}|\bar{a} \times \bar{b}|$$



$$\Longrightarrow D = \frac{|\bar{a} \times \bar{b}|}{|\bar{a}||\bar{b}|}$$

Jan 27-12:22 PM

Distance between two lines

$\mathcal{L}_1$: $x = 1 + t,\ y = -2 + 3t,\ z = 4 - t$ $\quad P_1(1,-2,4)$

$\mathcal{L}_2$: $x = 2t,\ y = 3 + t,\ z = -3 + 4t$ $\quad P_2(0,3,-3)$

$\bar{v}_1 = \langle 1,3,-1 \rangle$ is direction vector for $\mathcal{L}_1$

$\bar{v}_2 = \langle 2,1,4 \rangle$ " " " " "

These two lines are skew (Not parallel and can be embedded in two parallel planes.

$\bar{n} = \bar{v}_1 \times \bar{v}_2$ is $\perp$ to both $\bar{v}_1$ & $\bar{v}_2$

So a plane that contains $\mathcal{L}_1$ is.

$$\begin{array}{rr} \bar{v}_1 & \langle 1,3,-1 \rangle \\ \times & \langle 2,1,4 \rangle \\ \hline & \langle 13,-6,-5 \rangle = \bar{n} \end{array}$$

$(1,-2,4) \in \mathcal{L}_1$

$(0,3,-3) \in \mathcal{L}_2$

$\mathcal{P}_1$ $\quad 13(x-1)-6(y+2)-5(z-4)$ is a ~~plane~~ containing $\mathcal{L}_1$!

$\mathcal{P}_2$ $\quad 13(x-0)-6(y-3)-5(z+3)=0$ is plane containing $\mathcal{L}_2$!

$\mathcal{P}_1$ & $\mathcal{P}_2$ are parallel planes containing $\mathcal{L}_1$ & $\mathcal{L}_2$.

Distance from $\mathcal{L}_1$ to $\mathcal{L}_2$ =

Fact The distance between the two lines is the distance between the two planes.

& the distance between 2 PARALLEL planes is the distance between ANY point in $\mathcal{P}_2$ to the plane $\mathcal{P}_1$.

$\mathcal{P}_2$ $\quad 13(x-0)-6(y-3)-5(z+3)=0$

$(0,3,-3) \in \mathcal{P}_2$

Distance to $\mathcal{P}_1$

Hyperbolas & Ellipses

Example 10

$\mathcal{P}_1$ $\quad 13(x-1)-6(y+2)-5(z-4)$

$$\frac{|13(0-1)-6(3+2)-5(-3-4)|}{\sqrt{13^2+6^2+5^2}}$$

$$\frac{|-13-30+35|}{\sqrt{169+36+25}} = \frac{8}{\sqrt{230}}$$

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$x + y + z = 1$

$x + z = 0 \Rightarrow \boxed{x = -z}$

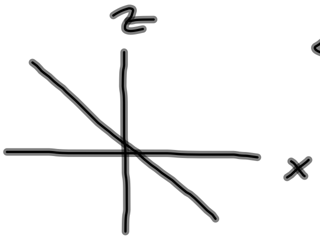
$-z + y + z = 1 \Rightarrow \boxed{y = 1}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 0 & -1 \end{array}\right]$$

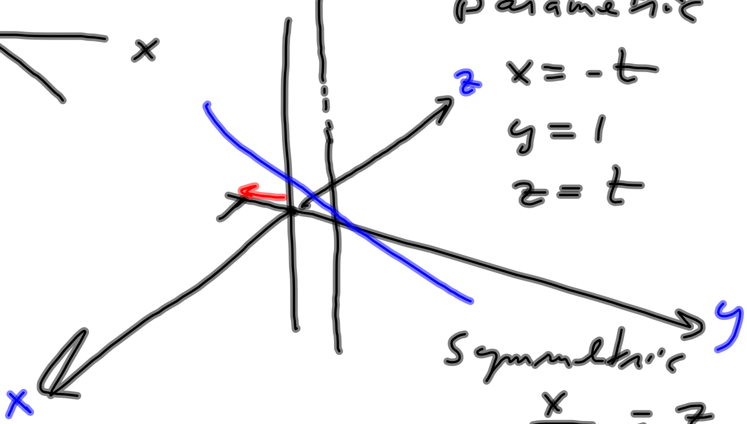
$$\begin{array}{cccc} x & y & z & c \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right]$$

$\boxed{\begin{array}{c} x + z = 0 \\ x = -z \end{array}}$

$y = 1$



$\langle -z, y, z \rangle$



parametric

$x = -t$

$y = 1$

$z = t$

Symmetric

$\frac{x}{-1} = z, \; y = 1$