§13.5 Finish.

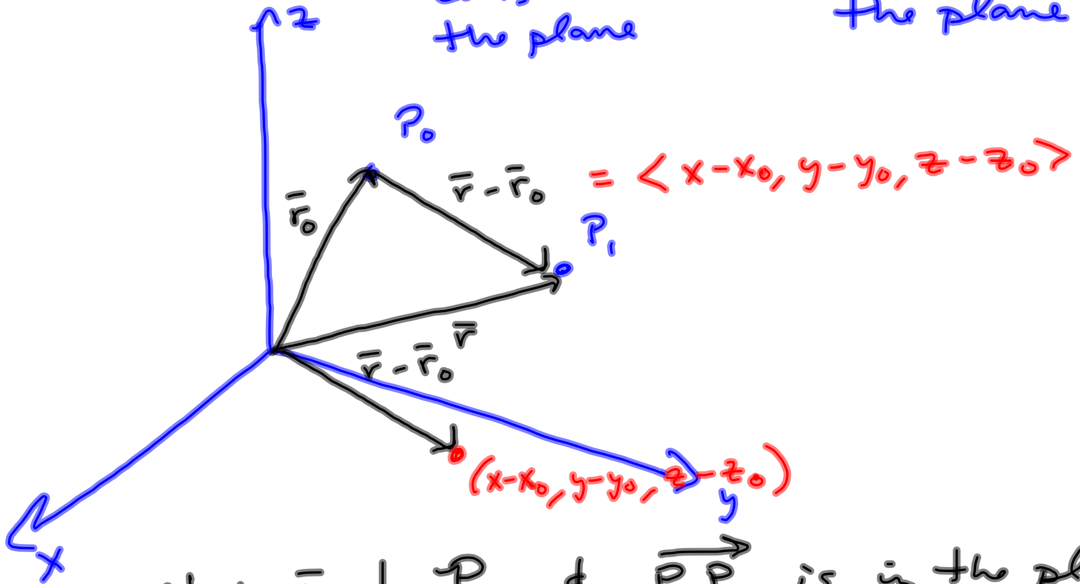
Let $\mathcal{P}$ be a plane $\bar{n} = \langle a, b, c \rangle$

Let $\bar{n}$ be orthogonal to the plane.

Let $P_0 = (x_0, y_0, z_0)$ & $P_1 = (x, y, z)$ lie in the plane.

$\bar{r}_0 = \langle x_0, y_0, z_0 \rangle$ & $\bar{r} = \langle x, y, z \rangle$

Then $\underbrace{\overrightarrow{P_0P_1}}_{\text{lies in the plane}} = \underbrace{\bar{r} - \bar{r}_0}_{\text{is parallel to the plane.}}$



Now, $\bar{n} \perp \mathcal{P}$ & $\overrightarrow{P_0P_1}$ is in the plane,

so $\bar{n} \bullet (\bar{r} - \bar{r}_0) = 0$

z

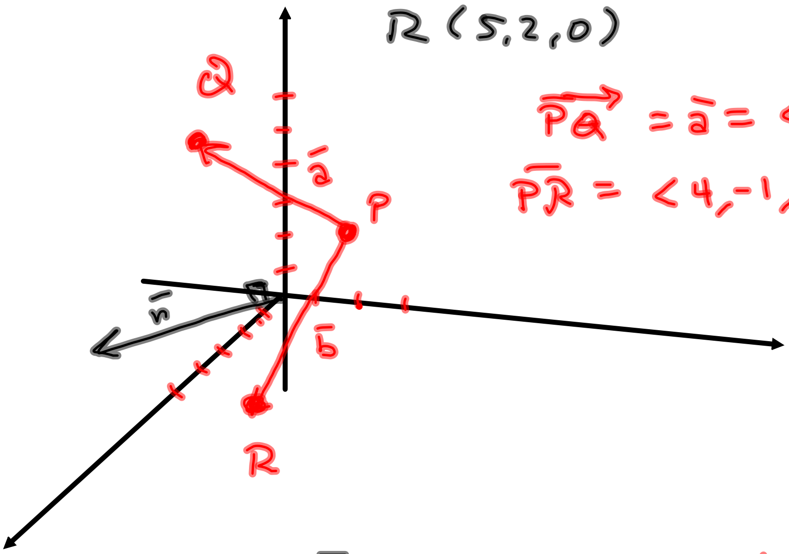
$= \langle a, b, c \rangle \bullet \langle x - x_0, y - y_0, z - z_0 \rangle$

$= a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

is an equation for the plane thru (x_0, y_0, z_0) and $\perp$ to $\langle a, b, c \rangle$

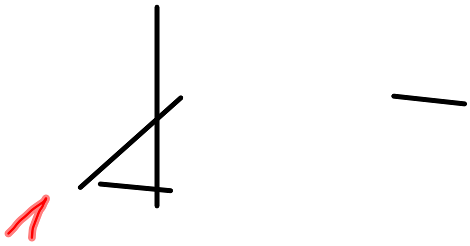
Jan 25-12:13 PM

Plane thru $P(1,3,2)$, $Q(3,-1,6)$, $R(5,2,0)$



$\overrightarrow{PQ} = \bar{a} = \langle 2,-4,4\rangle = \bar{a}$

$\overrightarrow{PR} = \langle 4,-1,-2\rangle = \bar{b}$



$\bar{n} = \bar{a} \times \bar{b}$

$$\begin{array}{r} \langle 2,-4,4\rangle \\ \times \langle 4,-1,-2\rangle \\ \hline \langle 12, +20, 14\rangle \end{array}$$

CRAP!

$\langle 12, +20, 14\rangle$ is $\perp$ to $\mathcal{P}$.

$\langle 6, 10, 7\rangle = \bar{n}$ is $\perp$ to $\mathcal{P}$.

$P(1,3,2)$

$6(x-1) + 10(y-3) + 7(z-2) = 0$

for ANY (x,y,z) in $\mathcal{P}$.

$\bar{c} = \langle x,y,z\rangle - \langle 1,3,2\rangle$ is parallel to $\mathcal{P}$.

$\bar{n} \bullet \bar{c} = 0$

How to tell if 2 planes are parallel?

$\bar{n}_1 \times \bar{n}_2 = \bar{0}$, because $\bar{n}_1$ & $\bar{n}_2$ are $\parallel$.

Representations of a line:

Vector: $\bar{r} + t\bar{v}$

Symmetric $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c} = t$

Parametric $x = x_0 + at$, $y = y_0 + bt$,
$z = z_0 + ct$

Plane "Regular": $a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$
$ax + by + cz + d = 0$

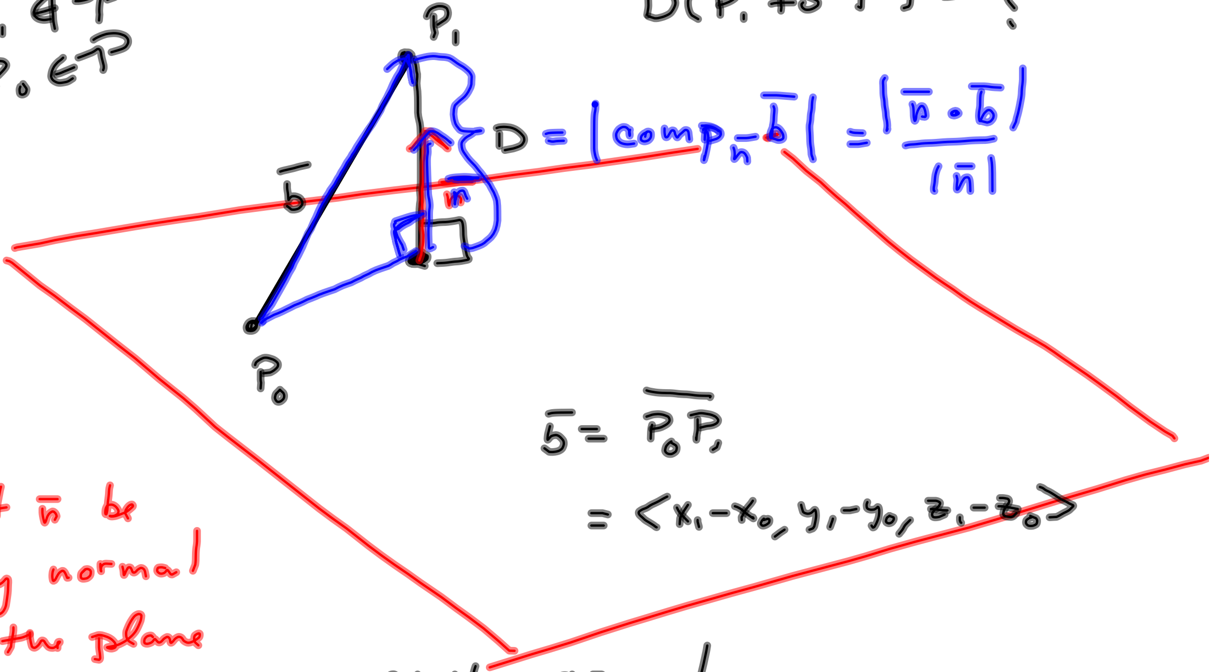
Jan 25-12:38 PM

Distance from pt. to plane.

$P_1 \notin \mathcal{P}$

$P_0 \in \mathcal{P}$

$D(P_1 \text{ to } \mathcal{P}) = ?$



$D = |\text{comp}_{\bar{n}} \bar{b}| = \dfrac{|\bar{n} \cdot \bar{b}|}{|\bar{n}|}$

$\bar{b} = \overline{P_0 P_1}$

$= \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

Let $\bar{n}$ be any normal to the plane

$ax + by + cz = d$

$\bar{n} = \langle a, b, c \rangle$

$P_0 = (x_0, y_0, z_0)$

$P_1 = (x_1, y_1, z_1)$

$\Rightarrow D(P_1 \text{ to } \mathcal{P})$

$= \dfrac{|\bar{n} \cdot \bar{b}|}{|\bar{n}|} =$

$$= \frac{|\langle a,b,c \rangle \cdot \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|a(x_1 - x_0) + b(y_1 - y_0) + c(z_1 - z_0)|}{\sqrt{a^2 + b^2 + c^2}} = D$$

Next time 13.6 & distance
from point to a line.
13.5 Monday

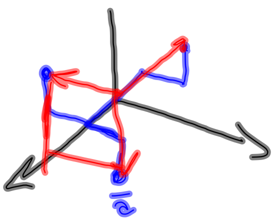
§13.4 #34

$\bar{a}\cdot(\bar{b}\times\bar{c}) = (\bar{a}\times\bar{b})\cdot\bar{c}$

$\bar{a} = \langle 1,1,-1\rangle$, $\bar{b} = \langle 1,-1,1\rangle$

$\bar{c} = \langle -1,1,1\rangle$

$\bar{a}\cdot(\bar{b}\times\bar{c})$



$$\begin{array}{r} \langle 1,-1,1\rangle \\ \times\ \langle -1,1,1\rangle \\ \hline \langle -2,-2,0\rangle \end{array}$$

$\bar{a}\cdot(\bar{b}\times\bar{c}) =$

$|\langle 1,1,-1\rangle\cdot\langle -2,-2,0\rangle|$

$= |-2-2| = |-4| = 4$

$\overrightarrow{PQ} = \langle 2,1,1\rangle = \bar{a}$

$\overrightarrow{PR} = \langle 1,-1,2\rangle = \bar{b}$

$\overrightarrow{PS}\ \ \langle 0,-2,3\rangle = \bar{c}$

PQ, PR, PS are adjacent

$\bar{a}\cdot(\bar{b}\times\bar{c})$

$a\cdot(\bar{c}\times\bar{b}) = -\bar{a}\cdot(\bar{b}\times\bar{c})$

$$\begin{array}{r} \langle 1,-1,2\rangle \\ \times\ \langle 0,-2,3\rangle \\ \hline \langle 1,-3,-2\rangle \end{array} = \bar{b}\times\bar{c}$$

$|\bar{a}\cdot(\bar{b}\times\bar{c})| = |\langle 2,1,1\rangle\cdot\langle 1,-3,-2\rangle|$

$= |2-3-2| = |-3|$

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4-6 = -2$

$\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} = 6-4 = +2$

Jan 25-12:59 PM