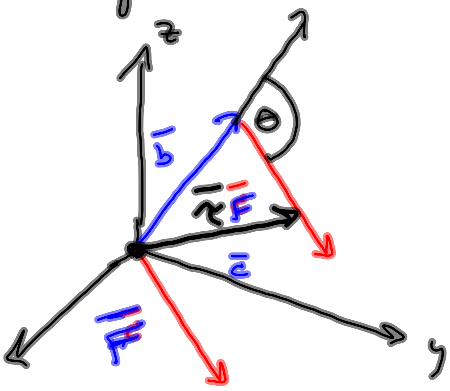
§13.5

Torque



$\bar\tau$ = Torque = $\bar b \times \bar F$

$\bar b$ is kinda in the yz-plane

$\bar F$ is " " " xy-plane

$\bar\tau$ is into the screen.

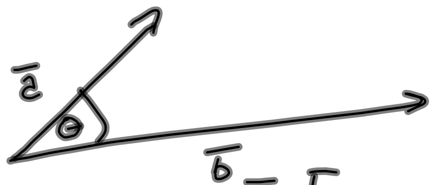
$|\bar\tau|$ is how much it tends to rotate.

$\bar\tau$ is the axis of revolution

Scalar Product $\bar{a}\cdot\bar{b} = \sum_{k=1}^{3} a_k b_k$

$|\bar{a}| = \sqrt{\bar{a}\cdot\bar{a}}$

$a_1^2 + a_2^2 + a_3^2 = \sum_{k=1}^{3} a_k a_k$



$\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|} \Longrightarrow \bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta$

component of $\bar{b}$ along $\bar{a}$ = $\text{comp}_{\bar{a}}\bar{b} = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|}$

projection .. .. $= \text{proj}_{\bar{a}}\bar{b} = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}|^2}\bar{a}$

$\bar{a}\times\bar{b}$

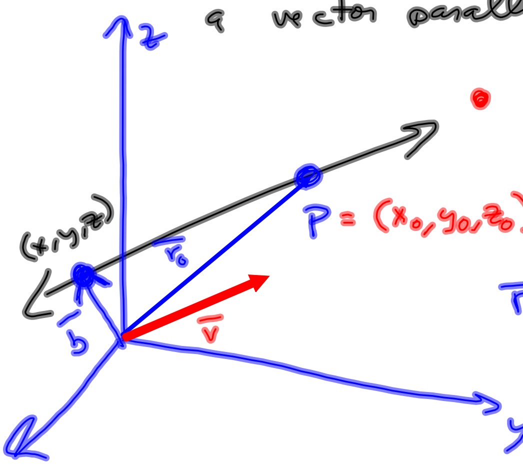
$$\begin{array}{r} \langle 1, 2, 3\rangle \\ \times\ \langle 1, -1, 2\rangle \\ \hline \langle 7, -2, 0\rangle \end{array}$$

$$\begin{array}{c} \langle a_1, a_2, a_3\rangle \\ \times\ \langle b_1, b_2, b_3\rangle \\ \hline \langle a_1(a_2b_3 - a_3b_2), -a_2(a_1b_3 - a_3b_1), a_3(a_1b_2 - a_2b_1)\rangle \end{array}$$

§13.5 Planes & Lines

A line : A point plus some multiple of a vector parallel to the line.

Position Vector



$P = (x_0, y_0, z_0) \longleftrightarrow \bar{r}_0 = \langle x_0, y_0, z_0 \rangle$

$\bar{r}_0 + t\bar{v}$

$\bar{v} = \langle v_1, v_2, v_3 \rangle$

Let $\langle x, y, z \rangle = \bar{b}$ be a position vector for (x, y, z)

$$\langle x, y, z \rangle = \bar{b} = \bar{r}_0 + t\bar{v} = \langle x_0 + tv_1, y_0 + tv_2, z_0 + tv_3 \rangle$$

$$\underbrace{x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3}$$

Parametric equations for the line thru $P(x_0, y_0, z_0)$ and parallel to $\bar{v} = \langle v_1, v_2, v_3 \rangle$
$= \langle a, b, c \rangle$ in book.

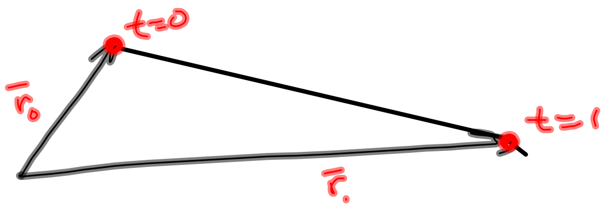
Symmetric Equations:

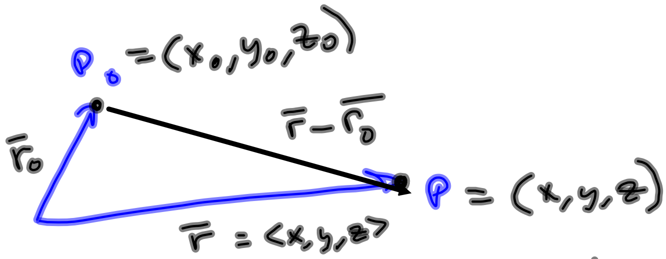
$$t = \frac{x - x_0}{v_1} = \frac{y - y_0}{v_2} = \frac{z - z_0}{v_3}$$

v_1, v_2, v_3 are direction #s for the line.

Line segment between $\bar{r}_0$ & $\bar{r}_1$

$$\bar{r}(t) = \bar{r}_0(1 - t) + \bar{r}_1 t$$
$$= (1 - t)\bar{r}_0 + t\bar{r}_1$$





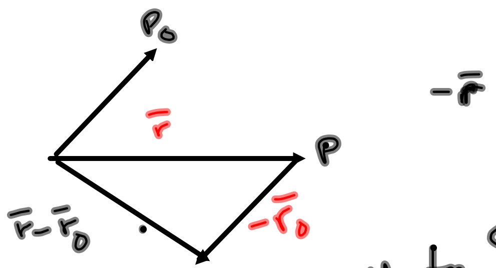
A plane is determined by a point in the plane and a vector orthogonal to the plane.

Let $\bar{n} = \langle a, b, c \rangle$ be orthogonal to the plane.

Then $\bar{r} - \bar{r}_0 = \overrightarrow{P_0 P}$

$\bar{n}$ is orthogonal to $\bar{r} - \bar{r}_0$

$\bar{r} - \bar{r}_0$ is parallel to the plane.



Then $\bar{n} \cdot (\bar{r} - \bar{r}_0) = 0$

Vector Eqns for the plane.

$$\bar{n} \cdot \bar{r} - \bar{n} \cdot \bar{r}_0 = 0$$
$$\bar{n} \cdot \bar{r} = \bar{n} \cdot \bar{r}_0$$

Another way of saying it:

$$\langle a, b, c \rangle \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0$$

$$\Rightarrow a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz + d = 0$$

(where $d = -ax_0 - by_0 - cz_0$)

(x, y, z) in plane satisfies these eq'ns.

$$ax + by + cz = -d$$

$\langle a, b, c \rangle$ is a normal vector to the plane.

Plane thru $P(1,3,2)$ & $Q(3,-1,6)$

$\bar{r}_0 = \langle 1,3,2\rangle$, $\bar{r} = \langle 3,-1,6\rangle$

$m(x-x_0) = y - y_0$

$$\begin{array}{r} \langle 1,3,2\rangle \\ \times \langle 3,-1,6\rangle \\ \hline \end{array}$$

$\bar{n} = \bar{r}_0 \times \bar{r} = \langle 20,0,-10\rangle$

Let $y=7$

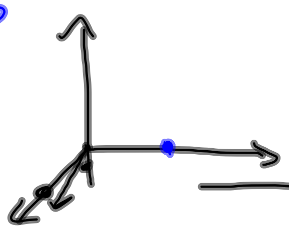
$\langle 2,0,-1\rangle$ works, too.

$\underline{2}(x-1)+\underline{0}(y-3)\,\underline{-1}(z-2) = 0$

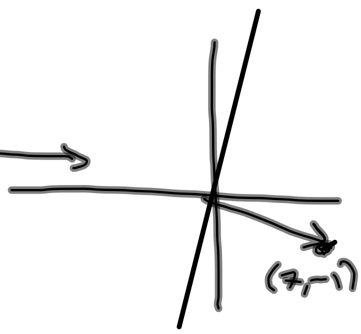
$2(0)+0-1(z-2)=0$

$y-y_0 = 7(x-x_0)$

$(y-y_0) = 7(x-x_0)$



$7(x-x_0) - (y-y_0) = 0$



$\langle 7,-1\rangle$

$m=7$

$m_{\perp} = -\frac{1}{7}$