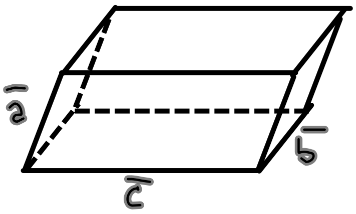
Recall

$$\bar{a} \cdot \bar{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle = \sum_{k=1}^{3} a_k b_k$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$\sum$'s cool for extra dimensions.

$$\langle a_1, a_2, \ldots, a_n \rangle \cdot \langle b_1, b_2, \ldots, b_n \rangle = \sum_{k=1}^{n} a_k b_k$$



parallelepiped volume $= |\bar{a} \cdot (\bar{b} \times \bar{c})|$

Angle between $\bar{a}$ & $\bar{b}$: $\cos\theta = \dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}||\bar{b}|}$

$\text{proj}_{\bar{a}} \bar{b}$ = vector projection =

$= \left(\text{comp}_{\bar{a}} \bar{b}\right) \dfrac{\bar{a}}{|\bar{a}|}$

$\text{comp}_{\bar{a}} \bar{b}$ = scalar projection = $\dfrac{\bar{a} \cdot \bar{b}}{|\bar{a}|}$

How big $\bar{b}$ is in the $\bar{a}$ direction

$\bar{a} \perp \bar{b}$ iff $\bar{a} \cdot \bar{b} = 0$

$\cdot : \mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ Dot maps into $\mathbb{R}$

New! $\times(\langle a_1, a_2, a_3 \rangle, \langle b_1, b_2, b_3 \rangle)$

$\times : \underline{\mathbb{R}^3} \times \underline{\mathbb{R}^3} \longrightarrow \underline{\mathbb{R}^3}$ $\times$ maps into $\mathbb{R}^3$

Distance function $D : \mathbb{R}^2 \times \mathbb{R}^2 \longrightarrow \mathbb{R}$

$$D\underbrace{\left((x_1, y_1), (x_2, y_2)\right)}_{\text{Input is a ordered pair of ordered pairs.}} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

pairs of ordered pairs live in $\mathbb{R}^2 \times \mathbb{R}^2$

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Determinant of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is

$(1)(4) - (3)(2) = \underline{\underline{-2}}$

Det: $\underline{\underline{M_{n\times n}}} \longrightarrow \mathbb{R}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \longrightarrow ad-bc = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

3x3 Determinant: <u>Expansion</u> by Minors.
↳ across 1^{st} row.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$= a(ei-fh)-b(di-gf)+c(dh-ge)$

$\bar{a} \times \bar{b}$ = Cross product of $\bar{a}$ & $\bar{b}$

= Vector " " " "

Dot product is the SCALAR product.
Cross product spits out a VECTOR.
This vector is orthogonal to both $\bar{a}$ & $\bar{b}$
Its MAGNITUDE, $|\bar{a} \times \bar{b}|$ is the area of the parallelogram defined by $\bar{a}$ & $\bar{b}$

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$$\bar{a} \times \bar{b} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

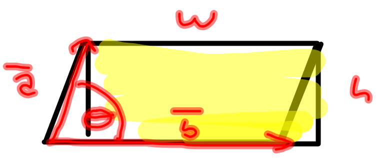
$$= \bar{i}\,(a_2b_3 - a_3b_2) - \bar{j}\,(a_1b_3 - a_3b_1) + \bar{k}\,(a_1b_2 - a_2b_1)$$

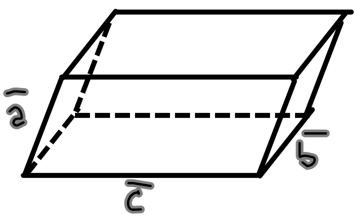
$$= \langle a_2b_3 - a_3b_2,\ -(a_1b_3 - a_3b_1),\ a_1b_2 - a_2b_1 \rangle$$

$$\begin{array}{r} \langle a_1,\ a_2,\ a_3 \rangle \\ \times\ \langle b_1,\ b_2,\ b_3 \rangle \\ \hline \langle a_2b_3 - a_3b_2,\ -(a_1b_3 - a_3b_1),\ a_1b_2 - a_2b_1 \rangle \end{array}$$

Easiest way to calculate.

Area of parallelogram





is the area of the rectangle

$= hw$

$= h|\bar{b}|$

$= |\bar{a}| \sin\theta \, |\bar{b}|$

$|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin\theta \geq 0$

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$$\boxed{|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}|\sin\theta} \geq 0 \, .$$

$$|\bar{a} \times \bar{b}|^2 = (\bar{a} \times \bar{b}) \cdot (\bar{a} \times \bar{b})$$

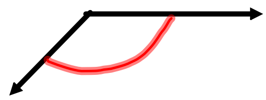
$$= \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle \cdot \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1 \rangle$$

$$= (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2$$

and then, a miracle occurs, and we have:

$$= |\bar{a}|^2|\bar{b}|^2 - (\bar{a} \cdot \bar{b})^2$$

$$= |\bar{a}|^2|\bar{b}|^2 - (|\bar{a}||\bar{b}|\cos\theta)^2$$



$$= |\bar{a}|^2|\bar{b}|^2 - |\bar{a}|^2|\bar{b}|^2(1 - \sin^2\theta)$$

$$= |\bar{a}|^2|\bar{b}|^2\sin^2\theta$$

$$\boxed{|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}|\sin\theta \qquad 0 \leq \theta \leq \pi}$$

$\bar{a} \parallel \bar{b}$ iff $\bar{a} \times \bar{b} = \bar{0}$

↳ parallel

$\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \sin\theta$

$= |\bar{a}||\bar{b}| \cdot 0$

$= 0$

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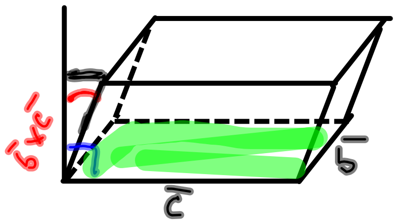
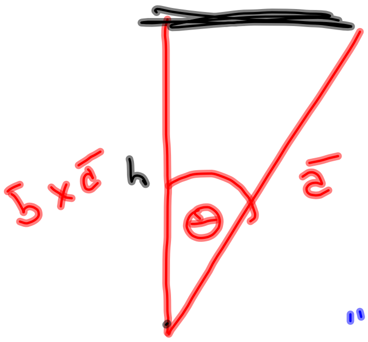


Volume of parallelepiped is $|\bar a \cdot (\bar b \times \bar c)| =$ triple scalar product



θ = angle between $\bar b \times \bar c$ & $\bar a$

height $= |\bar a| \cos\theta$

"length · width" · height

$|(\bar b \times \bar c)| |\bar a| \cos\theta$

$= |(\bar b \times \bar c)| |\bar a| \dfrac{|\bar a \cdot (\bar b \times \bar c)|}{|\bar a| |\bar b \times \bar c|}$

$= |\bar a \cdot (\bar b \times \bar c)|$