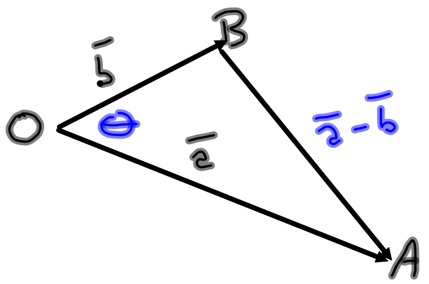
203 § 13.3 Dot Product

$\bar{a} - \bar{b} = \bar{a} + (-\bar{b})$



$\bar{a} = \langle a_1, a_2, a_3 \rangle$

$$|AB|^2 = |OA|^2 - 2|OA||OB|\cos\theta + |OB|^2$$

$$|\bar{a}-\bar{b}|^2 = |\bar{a}|^2 - 2|\bar{a}||\bar{b}|\cos\theta + |\bar{b}|^2$$

$$|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{a_1a_1 + a_2a_2 + a_3a_3}$$

$$= \sqrt{\bar{a}\cdot\bar{a}}$$

$$|\bar{a}|^2 = \bar{a}\cdot\bar{a}$$

$$\bar{a}\cdot\bar{b} = \bar{b}\cdot\bar{a}$$

$$\bar{a}\cdot(\bar{b}+\bar{c}) = \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c}$$

$$(\bar{a}-\bar{b})\cdot(\bar{a}-\bar{b}) = \bar{a}\cdot\bar{a} - 2\sqrt{\bar{a}\cdot\bar{a}}\sqrt{\bar{b}\cdot\bar{b}}\cos\theta + \bar{b}\cdot\bar{b}$$

$$\bar{a}\cdot\bar{a} - 2\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{b} = \bar{a}\cdot\bar{a} - 2\sqrt{\bar{a}\cdot\bar{a}}\sqrt{\bar{b}\cdot\bar{b}}\cos\theta + \bar{b}\cdot\bar{b}$$

$$-\cancel{2}\,\bar{a}\cdot\bar{b} = -\cancel{2}\sqrt{\bar{a}\cdot\bar{a}}\sqrt{\bar{b}\cdot\bar{b}}\cos\theta$$

$$\boxed{\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos\theta}$$

$$\boxed{\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}}$$

$\bar{a} = \langle a_1, a_2, a_3 \rangle$

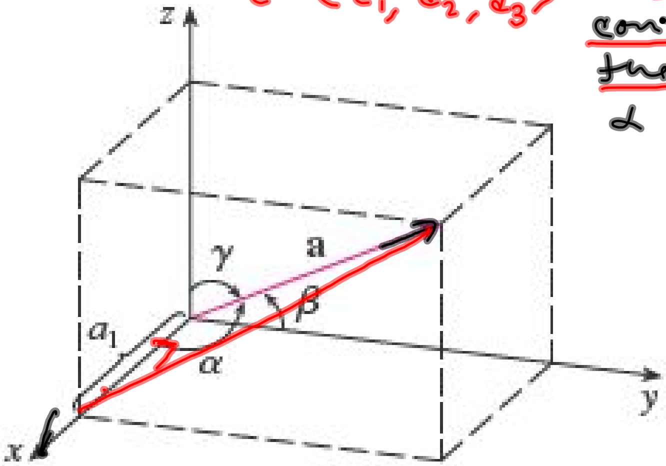
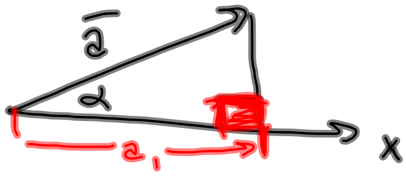


FIGURE 3

Consider the plane containing both $\bar{a}$ and the x-axis. The angle α lies in this plane.

In that plane:



$$\frac{a_1}{|\bar{a}|} = \cos \alpha$$

$$\cos\beta = \frac{a_2}{|\bar{a}|}, \quad \cos\gamma = \frac{a_3}{|\bar{a}|}$$

DIRECTION ANGLES AND DIRECTION COSINES

The **direction angles** of a nonzero vector **a** are the angles α, β, and γ (in the interval $[0, \pi]$) that **a** makes with the positive x-, y-, and z-axes. (See Figure 3.)

The cosines of these direction angles, $\cos\alpha$, $\cos\beta$, and $\cos\gamma$, are called the **direction cosines** of the vector **a**. Using Corollary 6 with **b** replaced by **i**, we obtain

$\langle a_1, a_2, a_3 \rangle \cdot \langle 1, 0, 0 \rangle = a_1$

8 $$\cos\alpha = \frac{\mathbf{a} \cdot \mathbf{i}}{|\mathbf{a}||\mathbf{i}|} = \frac{a_1}{|\mathbf{a}|}$$

9 $$\cos\beta = \frac{a_2}{|\mathbf{a}|} \qquad \cos\gamma = \frac{a_3}{|\mathbf{a}|}$$

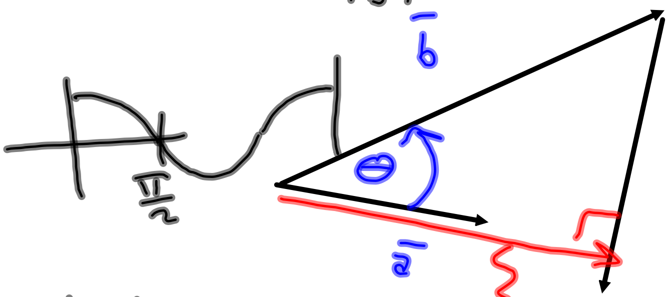
Projections

In the plane defined by two noncollinear vectors $\bar{a}$ & $\bar{b}$:

$\cos\theta = \dfrac{|\text{proj}_{\bar{a}}\bar{b}|}{|\bar{b}|}$

Shine a light from behind $\bar{b}$ perpendicular to $\bar{a}$

Then the projection of $\bar{b}$ onto $\bar{a}$ is the shadow cast by $\bar{b}$ on the line containing $\bar{a}$

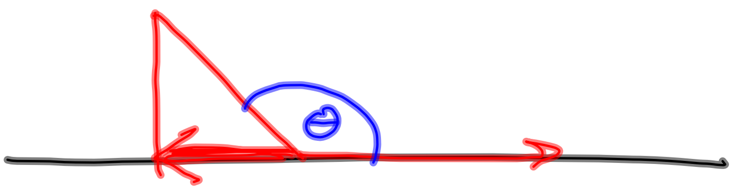


$\underbrace{|\bar{b}|\cos\theta}_{\text{Component of } \bar{b} \text{ along } \bar{a}} = |\text{proj}_{\bar{a}}\bar{b}|$

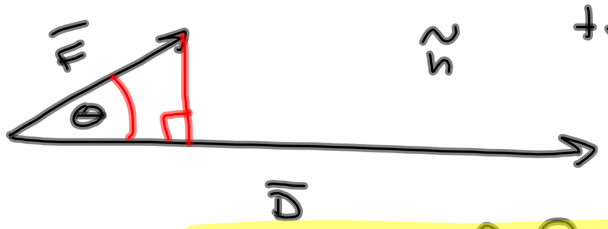
Let's give it direction:

$$\underbrace{\left(|\bar{b}|\cos\theta\right)}_{\text{length of the component of } \bar{b} \text{ in direction of } \bar{a}.}\underbrace{\frac{\bar{a}}{|\bar{a}|}}_{\text{unit vector in direction of } \bar{a}} = \text{proj}_{\bar{a}}\bar{b} = \cancel{|\bar{b}|}\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\cancel{|\bar{b}|}}\frac{\bar{a}}{|\bar{a}|}$$

$$= \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|^2}\bar{a}$$



Work



$\tilde{n}$ tilde

$\bar{u}$ $\hat{u}$

$\bar{u}$

work = ~~component of force~~ in direction of motion TIMES distance

$$|\bar{F}| \cos\theta \, |\bar{D}|$$

$$\cancel{|\bar{F}|} \, \frac{\bar{F}\cdot\bar{D}}{\cancel{|\bar{F}||\bar{D}|}} \, \cancel{|\bar{D}|} = \bar{F}\cdot\bar{D}$$

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