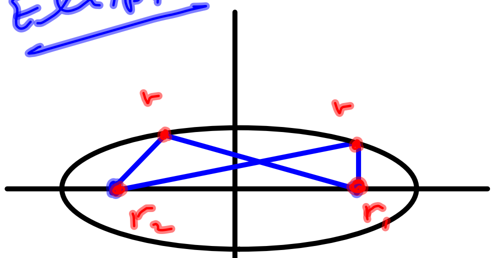
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$|r-r_1| + |r-r_2| = K$

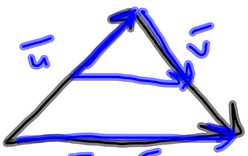
In line w/ geometric def'n of ellipse.

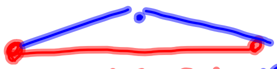
Ellipse




$|r-r_1| + |r-r_2| = K$

What if $K = |r_1 - r_2|$ ?



$|r_1 - r_2| = K$

$\frac{1}{2}\bar{u} + \frac{1}{2}\bar{v} = ?$

$= \frac{1}{2}(\bar{u}+\bar{v}) = \frac{1}{2}\bar{w}$

$\frac{1}{2}$ the length of $\bar{u}+\bar{v}$ & in the same direction.

# 13.2 VECTORS

Direction and magnitude.

Nose to tail

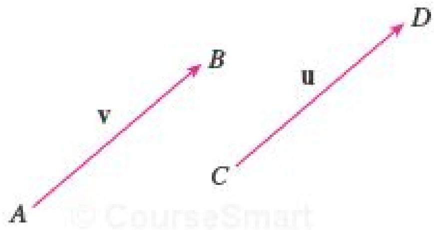





**FIGURE 1**
Equivalent vectors

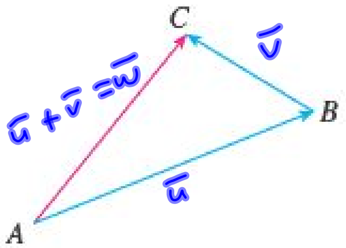
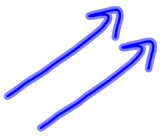



**FIGURE 2**



## COMBINING VECTORS

Suppose a particle moves from $A$ to $B$, so its displacement vector is $\overrightarrow{AB}$. Then the particle changes direction and moves from $B$ to $C$, with displacement vector $\overrightarrow{BC}$ as in Figure 2. The combined effect of these displacements is that the particle has moved from $A$ to $C$. The resulting displacement vector $\overrightarrow{AC}$ is called the *sum* of $\overrightarrow{AB}$ and $\overrightarrow{BC}$ and we write

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

**DEFINITION OF VECTOR ADDITION** If $\mathbf{u}$ and $\mathbf{v}$ are vectors positioned so the initial point of $\mathbf{v}$ is at the terminal point of $\mathbf{u}$, then the sum $\mathbf{u} + \mathbf{v}$ is the vector from the initial point of $\mathbf{u}$ to the terminal point of $\mathbf{v}$.



$(x,y,z)$ is a point in 3-space.
$\langle x,y,z\rangle$ " " position vector in 3-space.

"Nose to Tail" for addition.

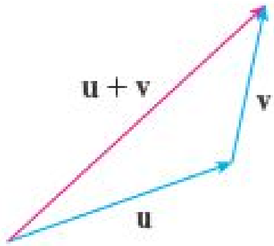



**FIGURE 3** The Triangle Law

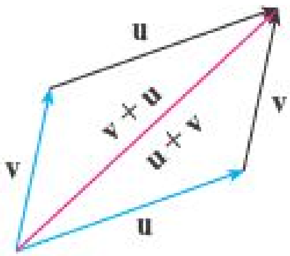
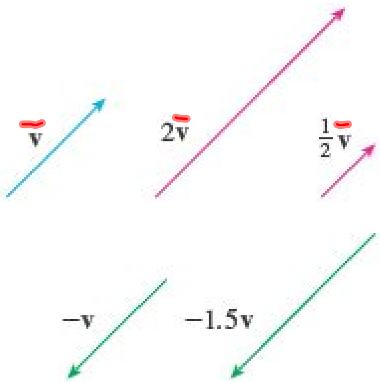



**FIGURE 4** The Parallelogram Law

Parallelogram Law illustrates commutativity of addition of vectors visually.

Scalar multiplication - multiplying a vector by a real number:

use the bar.



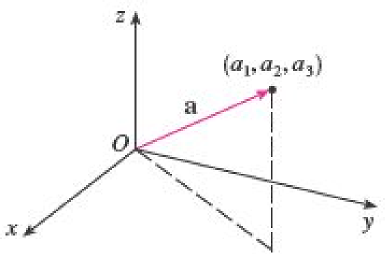

$3\langle 2,1,7\rangle =$

$\langle 6,3,21\rangle$

**FIGURE 7**
Scalar multiples of **v**

**DEFINITION OF SCALAR MULTIPLICATION** If $c$ is a scalar and $\mathbf{v}$ is a vector, then the **scalar multiple** $c\mathbf{v}$ is the vector whose length is $|c|$ times the length of $\mathbf{v}$ and whose direction is the same as $\mathbf{v}$ if $c > 0$ and is opposite to $\mathbf{v}$ if $c < 0$. If $c = 0$ or $\mathbf{v} = \mathbf{0}$, then $c\mathbf{v} = \mathbf{0}$.

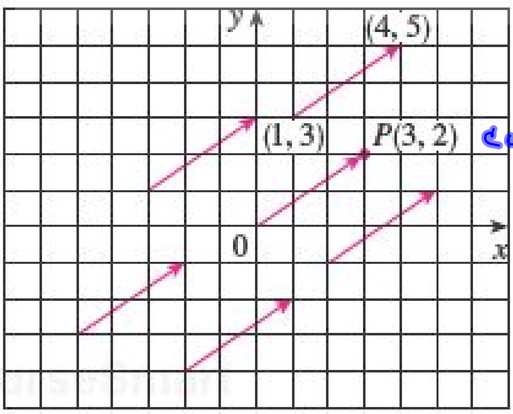
Difference of two vectors: $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$




$\mathbf{a} = \langle a_1, a_2, a_3 \rangle$

FIGURE 11

We use the notation $\langle a_1, a_2 \rangle$ for the ordered pair that refers to a vector so as not to confuse it with the ordered pair $(a_1, a_2)$ that refers to a point in the plane.




corresponds to <3, 2>, as do
All these directed line segment,

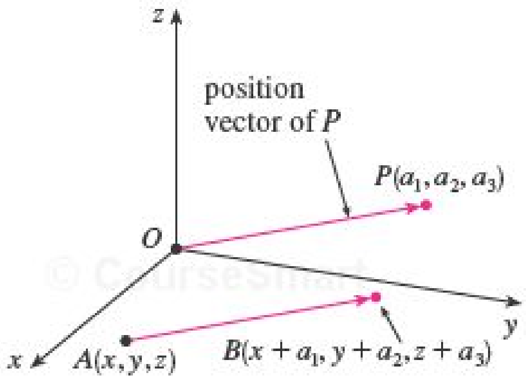
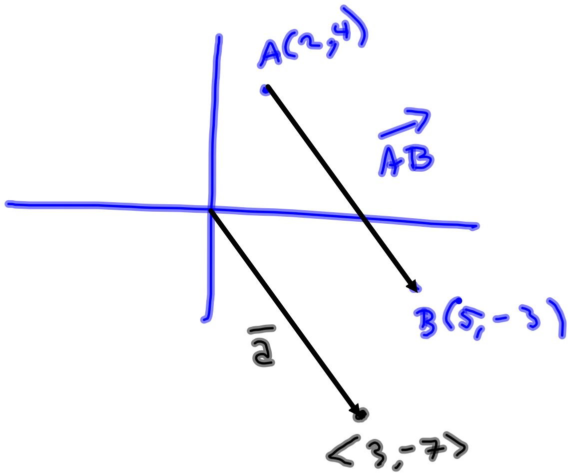



**FIGURE 13**
Representations of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$




**1** Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector $\mathbf{a}$ with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

The length of the two-dimensional vector $\mathbf{a} = \langle a_1, a_2 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$$

The length of the three-dimensional vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

If $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, then

$$\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2 \rangle \qquad \mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$$
$$c\mathbf{a} = \langle ca_1, ca_2 \rangle$$

Similarly, for three-dimensional vectors,



$$\langle a_1, a_2, a_3 \rangle + \langle b_1, b_2, b_3 \rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$$
$$\langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$$
$$c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$$

Properties of vectors.

$\bar{u} + \bar{v} = \bar{v} + \bar{u}$

$(\bar{u} + \bar{v}) + \bar{w} = \bar{u} + (\bar{v} + \bar{w})$

$\bar{a} + 0 =$ undefined

$\langle a_1, a_2, a_3 \rangle + 0$

$\bar{a} + \bar{0} = \bar{a}$

$\langle a_1, a_2, a_3 \rangle + \langle 0, 0, 0 \rangle = \langle a_1, a_2, a_3 \rangle$

$\bar{i} = \langle 1, 0, 0 \rangle$

$\bar{j} = \langle 0, 1, 0 \rangle$

$\bar{k} = \langle 0, 0, 1 \rangle$

Canonical basis for $\mathbb{R}^3$

Unit vectors $|\bar{i}| = \sqrt{1^2 + 0^2 + 0^2} = 1$

Mutually perpendicular.


Unit vectors in general

$\frac{1}{|\bar{u}|} \bar{u}$

**PROPERTIES OF VECTORS** If **a**, **b**, and **c** are vectors in $V_n$ and $c$ and $d$ are scalars, then

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$
3. $\mathbf{a} + \mathbf{0} = \mathbf{a}$
4. $\mathbf{a} + (-\mathbf{a}) = \mathbf{0}$
5. $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$
6. $(c + d)\mathbf{a} = c\mathbf{a} + d\mathbf{a}$
7. $(cd)\mathbf{a} = c(d\mathbf{a})$
8. $1\mathbf{a} = \mathbf{a}$

Standard Basis Vectors (Canonical Basis Vectors).

$$\mathbf{i} = \langle 1, 0, 0 \rangle \qquad \mathbf{j} = \langle 0, 1, 0 \rangle \qquad \mathbf{k} = \langle 0, 0, 1 \rangle$$

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, then we can write

$$\mathbf{a} = \langle a_1, a_2, a_3 \rangle = \langle a_1, 0, 0 \rangle + \langle 0, a_2, 0 \rangle + \langle 0, 0, a_3 \rangle$$
$$= a_1\langle 1, 0, 0 \rangle + a_2\langle 0, 1, 0 \rangle + a_3\langle 0, 0, 1 \rangle$$

2 $$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$$

A big deal in mathematics is generalizing 3-D to *n*-D.

Jan 13-10:45 AM

A **unit vector** is a vector whose length is 1. For instance, **i**, **j**, and **k** are all unit vectors. In general, if $\mathbf{a} \neq \mathbf{0}$, then the unit vector that has the same direction as **a** is



$$\boxed{4} \qquad \mathbf{u} = \frac{1}{|\mathbf{a}|}\mathbf{a} = \frac{\mathbf{a}}{|\mathbf{a}|}$$

$\bar{a} = \langle a_1, a_2, a_3 \rangle = \langle 1, 2, 3 \rangle$

$|\bar{a}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1+4+9} = \sqrt{14}$

unit vector in the direction of $\bar{a}$ is

$\frac{1}{|\bar{a}|}\bar{a} = \frac{1}{\sqrt{14}}\langle 1,2,3 \rangle = \langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \rangle$

$\left| \frac{\bar{a}}{|\bar{a}|} \right| = \sqrt{\left(\frac{1}{\sqrt{14}}\right)^2 + \left(\frac{2}{\sqrt{14}}\right)^2 + \left(\frac{3}{\sqrt{14}}\right)^2} = \sqrt{\frac{1}{14} + \frac{4}{14} + \frac{9}{14}}$

$= \sqrt{\frac{14}{14}} = 1$

Jan 13-10:46 AM

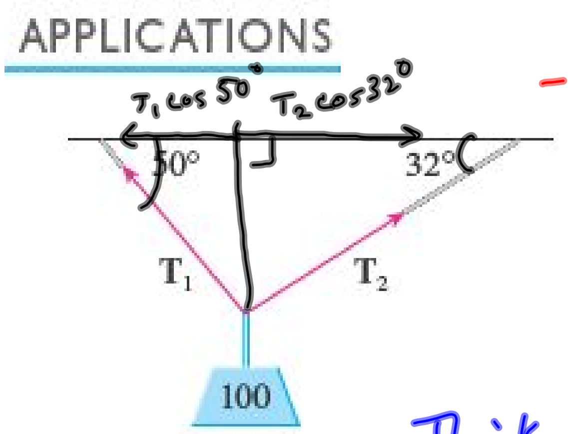



FIGURE 19

$$-|T_1|\cos(50^\circ) + |T_2|\cos(32^\circ) = 0$$

$$T_1 \sin(50^\circ) + T_2 \sin(32^\circ) = 100$$

$$T_1 \cos 50^\circ = -T_2 \cos 32^\circ$$

Think of $T_1$ & $T_2$ as positive forces.

Steve think better.

Jan 13-10:46 AM

**4.** Write each combination of vectors as a single vector.

(a) $\overrightarrow{PQ} + \overrightarrow{QR}$ $= \overrightarrow{PR}$

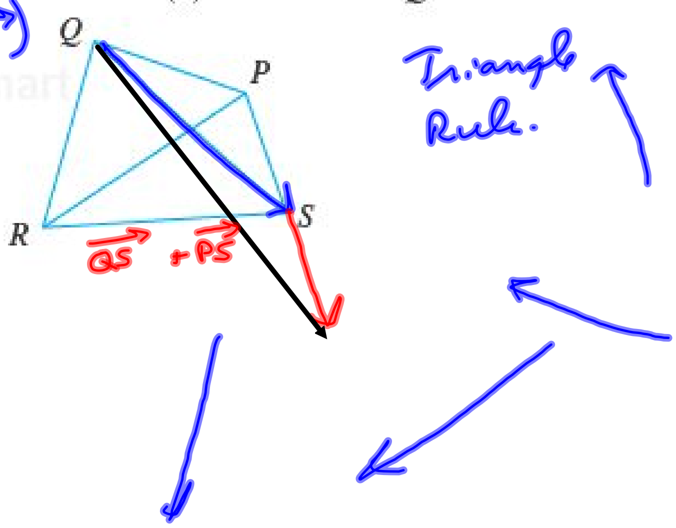
(b) $\overrightarrow{RP} + \overrightarrow{PS}$

(c) $\overrightarrow{QS} - \overrightarrow{PS}$

$= \overrightarrow{QS} + (-\overrightarrow{PS})$

$= \overrightarrow{QP}$

(d) $\overrightarrow{RS} + \overrightarrow{SP} + \overrightarrow{PQ}$ $= \overrightarrow{RS} - \overrightarrow{PS} + \overrightarrow{PQ}$



**7–12** Find a vector $\mathbf{a}$ with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

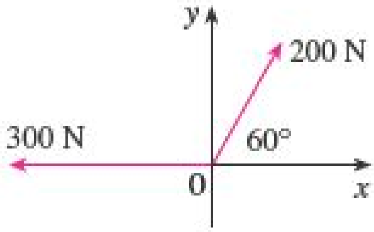
**10.** $A(2, 1)$, $B(0, 6)$

**12.** $A(4, 0, -2)$, $B(4, 2, 1)$

Jan 13-10:47 AM

**26.** If a child pulls a sled through the snow on a level path with a force of 50 N exerted at an angle of 38° above the horizontal, find the horizontal and vertical components of the force.

**29.**




To say a vector is determined by (an) other(s) is to say that you can "build" the vector by addition of multiples of the other(s).

**40.** Suppose that **a** and **b** are nonzero vectors that are not parallel and **c** is any vector in the plane determined by **a** and **b**. Give a geometric argument to show that **c** can be written as $\mathbf{c} = s\mathbf{a} + t\mathbf{b}$ for suitable scalars $s$ and $t$. Then give an argument using components.

Recall from College Algebra the geometric definition of an ellipse.

**42.** If $\mathbf{r} = \langle x, y \rangle$, $\mathbf{r}_1 = \langle x_1, y_1 \rangle$, and $\mathbf{r}_2 = \langle x_2, y_2 \rangle$, describe the set of all points $(x, y)$ such that $|\mathbf{r} - \mathbf{r}_1| + |\mathbf{r} - \mathbf{r}_2| = k$, where $k > |\mathbf{r}_1 - \mathbf{r}_2|$.

Some fairly interesting applications at the end of the exercises, too. :o)