MAT 203 | Final Test – Fall, 2020 | Name_______________________
100 Points | Covers Chapters 12-16 | NO GRAPHING CALCULATORS!!!

Due to lock-downs, this test will be take-home. I'm pleased we did all the other tests in traditional, hand-written fashion, but not this one. I'm still highly confident about the quality of work in this cohort.

1. Consider the two lines L_1 and L_2: $\begin{array}{l} L_1: \; x = 2+5t, y = 3-2t, \; z = 1+t \\ L_2: x = 5-3s, y = 1-s, z = s \end{array}$
   (a) (10 pts) Show that L_1 and L_2 are skew.
   (b) (10 pts) Find the distance between L_1 and L_2. For your final answer, I expect a simplified radical expression.

2. Consider the space curve $\mathbf{r} = \left\langle e^t \sin(t), e^t \cos(t), \sqrt{2}\, e^t \right\rangle = \left(e^t \sin(t)\right)\mathbf{i} + \left(e^t \cos(t)\right)\mathbf{j} + \left(\sqrt{2}\, e^t\right)\mathbf{k}$

   (a) (10 pts) Find the arc length function for the curve measured from the point $P\left(0, 1, \sqrt{2}\right)$ in the direction of increasing *t* and *then* re-paramatrize the curve with respect to arc length *s*. That is to say, find $t = t(s)$ as a function of *s* by finding $s = s(t)$ as a function of *t*, solve for *t*, and substitute that expression in for *t* in the expression for **r**(*t*) = **r**(*t*(*s*)).

   (b) (10 pts) Use your result in part (a) to find the point on the curve that is 4 units along the curve in the direction of increasing *t* from the point *P*. In other words, find **r**(*t*(4))!

   (c) (10 pts) Find the unit tangent **T**, unit normal **N**, and unit binormal **B** for **r**.

3. (10 pts) If $z = x^2 - xy + 3y^2$ and (x, y) changes from $(3, -1)$ to $(2.96, -0.95)$, compare the values of Δz and dz.

4. Verify Stokes' Theorem for the vector field $\mathbf{F}(x, y, z) = -2yz\,\mathbf{i} + y\,\mathbf{j} + 3x\,\mathbf{k} = \langle -2yz, y, 3x \rangle$ and the paraboloid $S = \left\{(x, y, z) \mid z = 4 - x^2 - y^2,\ 0 \le z \le 4\right\}$, oriented upward.
   (a) (10 points) ... for the line integral.
   (b) (10 points) ... for the surface integral.

5. A fluid has density $\rho = 5\ \dfrac{kg}{m^3}$ and flows with velocity $\mathbf{v} = x\,\mathbf{i} + y^2\,\mathbf{j} + 2yz\,\mathbf{k} = \left\langle x, y^2, 2yz \right\rangle$, where *x*, *y* and *z* are measured in meters, and the components of **v** in meters per second. Find the rate of (mass) flow (i.e., flux) through the cylinder $S = \left\{(x, y, z) \mid x^2 + y^2 = 4,\ 0 \le z \le 1\right\}$ in two different ways:

   (a) (10 pts) Compute the double integral of the normal component of the velocity field over *S*:
   $$\iint_S \rho \mathbf{v} \cdot d\mathbf{S} = \rho \iint_S \mathbf{v} \cdot d\mathbf{S}$$
   (b) (10 pts) Use the Divergence Theorem.