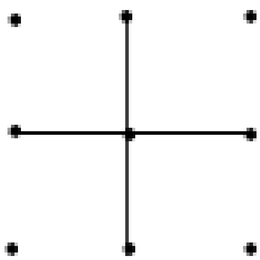
MAT 203 Test 4 – Fall, 2020 Name________________________

60 Points Covers Sections 16.1-5 NO GRAPHING CALCULATORS!!!

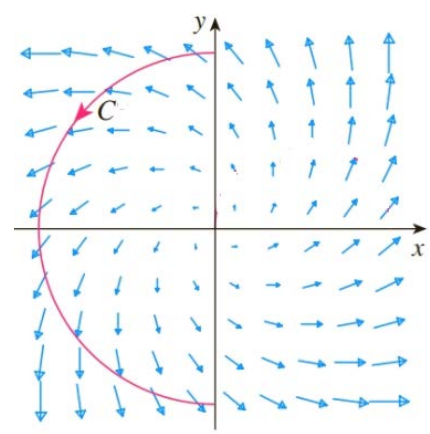
Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes! Resist the temptation to finish a problem on the page it started. When you get near the bottom, start another page.

We're mostly looking for integral setups, so read the instructions, carefully, so as not to waste time on messy integrals, unnecessarily.

1. (10 pts) Sketch the vector field $\mathbf{F} = \langle x+y, x-y \rangle$ in the space provided. There are 9 points shown, corresponding to points on the axes or one unit away from the axes.



2. (10 pts) Is $\int_C \mathbf{F} \bullet d\mathbf{r}$ positive, negative or zero, if $\mathbf{F}$ is the vector field and C is the curve shown on the right?



3. (20 pts) Evaluate (Yes, evaluate.) the line integral $\int_C \left(x^2+y^2+z^2\right) ds$, where C is the spiral $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), t \rangle$, $0 \le t \le 4\pi$.

4. Let $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy+2z)\,\mathbf{k} = \langle yz, xz, xy+2z \rangle$.

(a) (10 pts) Show that $\mathbf{F}$ is conservative.

(b) (10 pts) Find f such that $\nabla f = \mathbf{F}$

(c) (10 pts) Use part (b) to evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$, where C is the spiral given by the vector-valued function $\mathbf{r}(t) = (3+\cos(t))\mathbf{i} + (2+\sin(t))\mathbf{j} + (3t)\mathbf{k} = \langle 3+\cos(t), 2+\sin(t), 3t \rangle$, $0 \le t \le 2\pi$.

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(d) (10 pts) Use previous work to evaluate $\int_{\hat{C}} \mathbf{F} \bullet d\mathbf{r}$, where $\hat{C}$ is the curve given by

$\langle \sin(t), 4\cos(t), 3\sin(t)\cos(t) \rangle, 0 \le t \le 2\pi$

5. (20 pts) Use Green's Theorem to evaluate $\int_C \left(\sin(x) - 4xy^2\right)dx + \left(\cos(y) - 2x^2 y\right)dy$, where C is the triangle with vertices $(0,0), (5,0)$, and $(5,3)$, traversed *clock-wise*.

6. (10 pts) Is there a vector field **F** such that $curl(\mathbf{F}) = \langle 3\sin(z) + 2x\sin(xz), -3y\sin(z), -2z\sin(xz) - x \rangle$?

Bonus Section: Answer up to 15 points' worth.

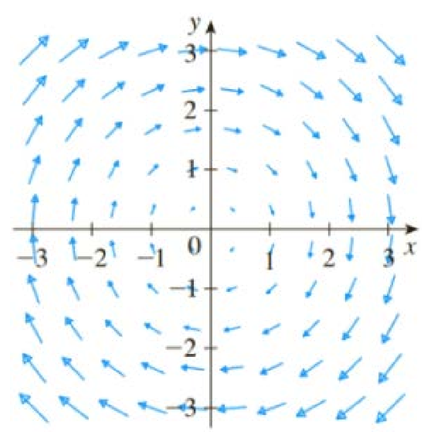
1. (Quick-hitter 5 pts) Evaluate the line integral $\int_C xy^2 dx + x^2 y\, dy$ around the circle $\left(x - \frac{3}{7}\right)^2 + \left(y + \frac{32}{\pi}\right)^2 = 13$

   (a) directly and (b) using Green's Theorem. You may cite facts or theorems, to save work, provided your arguments are cogent (convincing).

2. (2 pts each) Write 5 other iterated integrals that are equivalent to the iterated integral
   $\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z)\, dz\, dy\, dx$

3. (5 pts) Find the distance from the point $P(1,3,2)$ to the plane $3x - 2y + z = 7$.

4. (5 points) Find $\frac{\partial f}{\partial x}$ for $f(x, y) = \int_{x^2 - 2x}^{1} \frac{\sqrt[3]{\sin^2(t) + \pi}}{\cos(2t) + 2} dt$.



5. (5 pts) Explain the meaning of the equation

   $\int_{\partial D} \mathbf{F} \bullet d\mathbf{r} = \iint_D curl(\mathbf{F}) \bullet \mathbf{k}\, dA$ in words.

6. (5 pts) In what direction does the curl point for the vector field **F** in the graph on the right?