MAT 203
60 Points

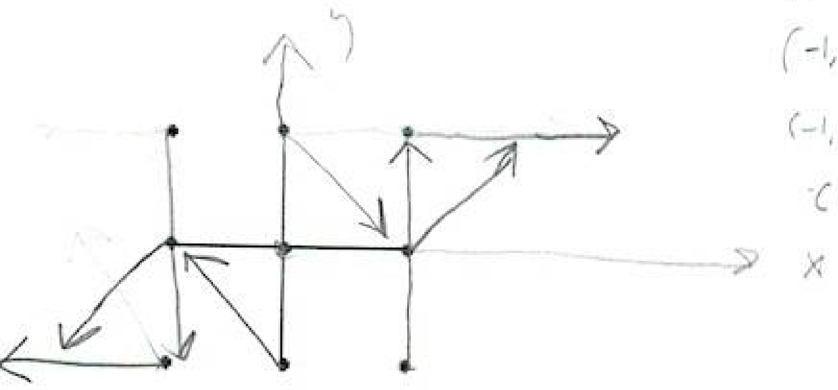
Test 4 – Fall, 2020
Covers Sections 16.1-5

Name KEY
NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes! Resist the temptation to finish a problem on the page it started. When you get near the bottom, start another page.

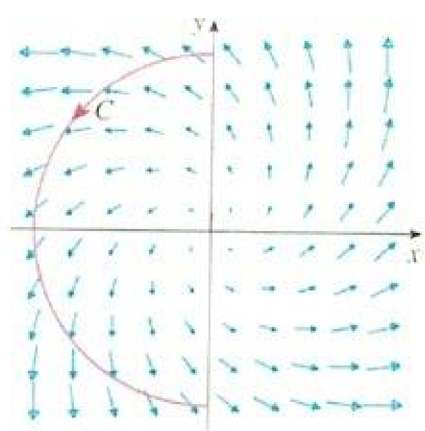
We're mostly looking for integral setups, so read the instructions, carefully, so as not to waste time on messy integrals, unnecessarily.

1. (10 pts) Sketch the vector field $\mathbf{F} = \langle x+y, x-y \rangle$ in the space provided. There are 9 points shown, corresponding to points on the axes or one unit away from the axes.



$(-1,1)$ $\langle -1+1, -1-1 \rangle$
$(-1,0)$ $\langle -1, -1 \rangle$
$(1,0)$ $\langle 1, 1 \rangle$
$(0,-1)$ $\langle -1, 1 \rangle$
$(1,-1)$ $\langle 0, 2 \rangle$
$(0,-1)$ $\langle -1, 1 \rangle$

2. (10 pts) Is $\int_C \mathbf{F} \bullet d\mathbf{r}$ positive, negative or zero, if $\mathbf{F}$ is the vector field and C is the curve shown on the right?



3. (20 pts) Evaluate (Yes, evaluate.) the line integral $\int_C (x^2+y^2+z^2)\,ds$, where C is the spiral $\mathbf{r}(t) = \langle \cos(2t), \sin(2t), t \rangle$, $0 \le t \le 4\pi$.

4. Let $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + (xy+2z)\mathbf{k} = \langle yz, xz, xy+2z \rangle$.

(a) (10 pts) Show that $\mathbf{F}$ is conservative.

(b) (10 pts) Find f such that $\nabla f = \mathbf{F}$

(c) (10 pts) Use part (b) to evaluate the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$, where C is the spiral given by the vector-valued function $\mathbf{r}(t) = (3+\cos(t))\mathbf{i} + (2+\sin(t))\mathbf{j} + (3t)\mathbf{k} = \langle 3+\cos(t), 2+\sin(t), 3t \rangle$, $0 \le t \le 2\pi$.

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(d) *(10 pts) Use previous work to evaluate* $\int_{\hat{C}} \mathbf{F} \bullet d\mathbf{r}$, *where* $\hat{C}$ *is the curve given by*

$\langle \sin(t), 4\cos(t), 3\sin(t)\cos(t) \rangle, 0 \le t \le 2\pi$

5. (20 pts) Use Green's Theorem to evaluate $\int_C (\sin(x) - 4xy^2)\,dx + (\cos(y) - 2x^2y)\,dy$, where C is the triangle with vertices $(0,0), (5,0)$, and $(5,3)$, traversed *clock-wise*.

6. (10 pts) Is there a vector field **F** such that $curl(\mathbf{F}) = \langle 3\sin(z) + 2x\sin(xz), -3y\sin(z), -2z\sin(xz) - x \rangle$?

Bonus Section: Answer up to 15 points' worth.

1. (Quick-hitter 5 pts) Evaluate the line integral $\int_C xy^2\,dx + x^2y\,dy$ around the circle $\left(x - \frac{3}{7}\right)^2 + \left(y + \frac{32}{\pi}\right)^2 = 13$

   (a) directly and (b) using Green's Theorem. You may cite facts or theorems, to save work, provided you do so in cogent fashion.

2. (2 pts each) Write 5 other iterated integrals that are equivalent to the iterated integral

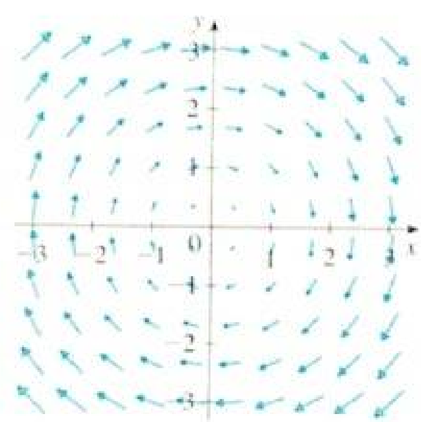
   $\int_0^1 \int_0^{x^2} \int_0^y f(x,y,z)\,dz\,dy\,dx$

3. (5 pts) Find the distance from the point $P(1,3,2)$ to the plane $3x - 2y + z = 7$.

4. (5 points) Find $\frac{\partial f}{\partial x}$ for $f(x,y) = \int_{x^2-2x}^{1} \frac{\sqrt[3]{\sin^2(t) + \pi}}{\cos(2t) + 2}\,dt$.

5. (5 pts) Explain the meaning of the equation

   $\int_{\partial D} \mathbf{F} \bullet d\mathbf{r} = \iint_D curl(\mathbf{F}) \bullet \mathbf{k}\,dA$ in words.

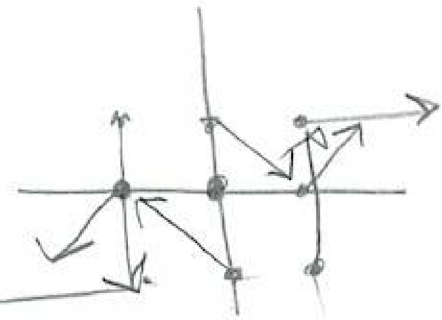


6. (5 pts) In what direction does the curl point for the vector field **F** in the graph on the right?

203 TEST 4 FALL '20

① $\bar{F} = \langle x+y,\ x-y \rangle$

10pts



10pts

② $\int_C \bar{F}\cdot d\bar{r} > 0$

③ 20pts $\int_C (x^2+y^2+z^2)\,ds$

$\bar{r} = \langle \cos(2t), \sin(2t), t \rangle \quad 0 \le t \le 4\pi$

$\bar{r}' = \langle -2\sin(2t), 2\cos(2t), 1 \rangle \Longrightarrow$

$\|\bar{r}'\| = \sqrt{4\sin^2(2t) + 4\cos^2(2t) + 1^2} = \sqrt{4+1} = \sqrt{5}$

$\Longrightarrow \int_C (x^2+y^2+z^2)\,ds = \sqrt{5}\int_0^{4\pi} \left(4\cos^2(2t) + 4\sin^2(2t) + t^2\right) dt$

$= \sqrt{5}\int_0^{4\pi} (4+t^2)\,dt = \sqrt{5}\left[4t + \frac{t^3}{3}\right]_0^{4\pi}$

$= \boxed{16\sqrt{5}\,\pi + \frac{64\sqrt{5}\,\pi^3}{3}}$

203 TEST 4 FALL '20

(4) $\bar{F} = \langle yz, xz, xy+2z \rangle$

$\bar{F}$ is infinitely continuously differentiable.

(a) curl $\bar{F}$:

(10 pts)

$$\begin{array}{c} x \quad y \quad z \qquad x \quad y \\ \times \langle yz, xz, xy+2z \rangle \; yz, xz \\ \hline \langle x-x, y-y, z-z \rangle = \bar{0} \Longrightarrow \end{array}$$

$\bar{F}$ is conservative.

(b) $f_x = yz \Longrightarrow f = xyz + g(y,z) \Longrightarrow$

$f_y = xz = xz + g_y(y,z) \Longrightarrow g_y = 0 \Longrightarrow g = g(z) \Longrightarrow$

$f_z = xy + 2z = xy + g'(z) \Longrightarrow$

(10 pts) $g'(z) = 2z \Longrightarrow g(z) = z^2 \Longrightarrow$

$$\boxed{f = xyz + z^2}$$

(c) (10 pts) $\int_C \bar{F} \cdot dr$ where C is given by

$\bar{r}(t) = \langle 3+\cos t, 2+\sin(t), 3t \rangle, \quad 0 \le t \le 2\pi$

$f(\bar{r}(2\pi)) - f(\bar{r}(0))$

$= (3+\cos 2\pi)(2+\sin 2\pi)(3(2\pi)) + (3 \cdot 2\pi)^2$

$- (3+\cos(0))(2+\sin(0))(3(0)) + (3(0))$

$= (4)(2)(6\pi) + 36\pi^2 = \boxed{48\pi + 36\pi^2}$

203 TY FALL'20

#4 cnt'd

(d) $\hat{C}$ given by $\bar{r} = \langle \sin(t), 4\cos(t)\ , 3\sin(t)\cos t \rangle$

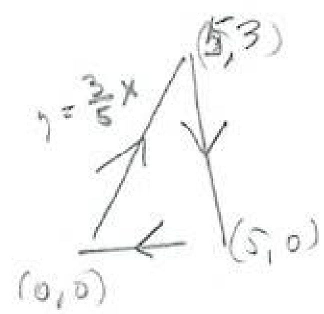
$\bar{r}(2\pi) = \langle 0, 2, 0 \rangle$ $\longrightarrow$ $\hat{C}$ is closed curve. Also simple.

$\bar{r}(0) = \langle 0, 2, 0 \rangle$

$\int_{\hat{C}} \bar{F} \cdot d\bar{r} = 0$, as $\bar{F}$ is conservative.

(5) (20 pts) $\int_C (\sin x - 4xy^2)\,dx + (\cos y - 2x^2y)\,dy$

$= \iint_D (-4xy + 8xy)\,dA = \int_0^5 \int_0 4xy\,dy\,dx$



$= \int_0^5 \left[2xy^2\right]_0^{\frac{3}{5}x} dx = \int_0^5 2x\left(\frac{3x}{5}\right)^2 dx = \int_0^5 \frac{18}{25} x^3\,dx$

$= \frac{9}{50} x^4 \Big]_0^5 = \frac{9}{50}(5)^4 = \frac{9}{2}(5)^2 = \frac{225}{2}$

Wait! Clockwise! So $\boxed{-\frac{225}{2}}$

is final answer.

203 T4 FALL '20

(6) (10pts) $\bar{G} = \langle 3\sin z + 2x\sin(xz), -3y\sin(z), -2z\sin(xz) - x \rangle$

$\Rightarrow \operatorname{div}\bar{G} = 2\sin(xz) + 2xz\cos(xz) - 3\sin(z) - 2\sin(xz) - 2xz\cos(xz) \neq 0 \longrightarrow \boxed{NO}$

$= -3\sin(z) \neq 0$

(B1) $\int_C xy^2\,dx + x^2y\,dy$ around $(x-\frac{3}{7})^2 + (y+\frac{32}{\pi})^2 = 13$

$Q_x - P_y = 2xy - 2xy = 0 \longrightarrow \bar{F}$ conservative

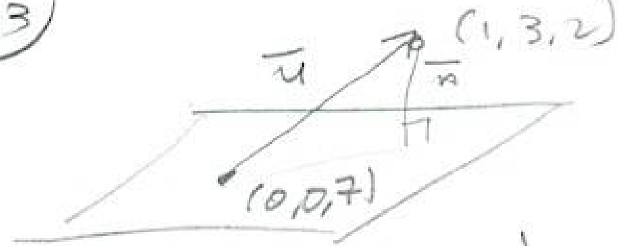
$\Rightarrow \int_C \bar{F}\cdot d\bar{r} = \int_C \langle xy^2, x^2y\rangle \cdot \langle x', y'\rangle\,dt$

$= \int_C xy^2\,dx + x^2y\,dy = 0$

(B2)

203 T4 FALL '20

B3



$$d = |\text{comp}_{\bar n}\, \bar u| = \frac{|\bar u \cdot \bar n|}{\|\bar n\|} = \frac{|\langle 1,3,-5\rangle \cdot \langle 3,-2,1\rangle|}{\sqrt{14}}$$

$$= \frac{|3-6-5|}{\sqrt{14}} = \frac{|-8|}{\sqrt{14}} = \boxed{\frac{8}{\sqrt{14}} = \frac{8\sqrt{14}}{14} = \frac{4\sqrt{14}}{7}}$$

B4 $$f(x,y) = \int_{x^2-2x}^{13} \frac{\sqrt{\sin^2(t)+\pi}}{\cos(2t)+2}\,dt \Longrightarrow$$

$$\boxed{f_x = -\left(\frac{\sqrt{\sin^2(x^2-2x)+\pi}}{\cos(2(x^2-2x))+2}\right)(2x-2)}$$

B5 $\int_{\partial D} \bar F \cdot d\bar r = \iint_D \text{curl}(\bar F)\cdot \bar k \, dA$ means the line integral of the tangential component of $\bar F$ along the curve $C = \partial D$ is equal to the double integral of the vertical component of the curl of $\bar F$ over the region D enclosed by $C = \partial D$.

203 TY Fall '20

(6) Curl $\vec{F}$ points into the page by the right-hand rule