

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

Make sure you have everything set up before doing any messy calculations. Draw more pictures than you want to.

1. (20 pts) Use Fubini to calculate the double integral  $\iint_R \cos(\theta)\sin(\phi) dA$ , where  $R$  is the rectangle

$$\left\{ (\theta, \phi) \mid 0 \leq \theta \leq \frac{\pi}{3}, 0 \leq \phi \leq \frac{\pi}{4} \right\}.$$

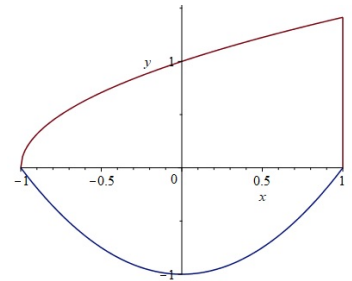
2. Polar and Cylindrical Coordinates.

- a. (20 pts) Use polar coordinates and a double integral to find the total mass of the lamina that is the portion of a circle of radius  $r = 3$  centered at the origin that lies in the first quadrant, if the mass density function  $\rho(x, y)$  at  $Q(x, y)$  is proportional to the square of  $Q$ 's distance from the origin.
- b. (5 pts **bonus**) Write a double integral for the previous problem in *rectangular* coordinates.
- c. (10 pts) Use cylindrical coordinates and a triple integral to find the volume of the solid bounded by the paraboloid  $z = 9 - x^2 - y^2$  in the first octant.
- d. (5 pts **bonus**) Write a triple integral for the previous problem in *rectangular* coordinates.

3. SET UP (Do Not Evaluate) the double integral  $\iint_R x dA$  over the region  $R$

bounded by  $y = x^2 - 1$ ,  $y = \sqrt{x+1}$ ,  $x = 1$ . Do this by viewing  $R$  as a...

- a. (20 pts) ... Type I region.
- b. (10 pts) ... Type II region.



4. (20 pts) Evaluate  $\iiint_R \sqrt{x^2 + y^2 + z^2} dV$ , where  $R$  lies above the cone  $z = \sqrt{x^2 + y^2}$  and between the spheres  $x^2 + y^2 + z^2 = 1$  and  $x^2 + y^2 + z^2 = 4$ . Use spherical coordinates.

**Bonus Section for sit-down test:**

1. (10 pts) Find the Jacobian of a change of variables that maps the parallelogram bounded by the inequalities  $6 \leq 3x + 2y \leq 12$ ,  $15 \leq 5x - 3y \leq 30$  into a nice rectangle in the  $uv$ -plane.
2. (10 pts) Write as many of the 5 other iterated integrals that are equivalent to the iterated integral

$$\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx$$

as you can find. 2 points apiece.