MAT 203 Test 2 – Fall, 2020 Name_______________________

100 Points. 70% of Test 2 Covers Chapter 14 NO GRAPHING CALCULATORS!!!

Do all your work and submit answers with your work, on the separate paper provided. Organize your work for efficient grading and feedback. Leave a margin, especially in the top left, where the staple goes!

1. (10 pts) Find and graph the domain of $f(x,y)=\sqrt{x+1}+\sqrt{25-x^2}-\sqrt{y^2-1}$.

2. (10 pts) Find the first partials f_x and f_y for $f(x,y)=\left(2x^2-3x^2y+5y^4\right)^3$.

3. Find $\frac{\partial z}{\partial x}$ for the equation $2x^2yz^2=2x^2y^3z$ in 2 ways:

   a. (5 pts) Use implicit differentiation, holding *y* constant and treating *z* as an implicit function of *x*.

   b. (5 pts) Form a function $F(x,y,z)$ and find $\frac{\partial z}{\partial x}$ for the level surface $F(x,y,z)=0$.

4. Let $f(x,y)=2x^2y^3-\frac{4}{\pi}\sin(\pi xy)$.

   a. (5 pts) Find an equation of the tangent plane to f at the point $(1,\,2,\,f(1,2))=(1,\,2,16)$.

   b. (5 pts) Use the linearization at $(1,\,2,16)$ to approximate $f(1.2,\,1.9)$

   c. (5 pts) Find the actual value of $f(1.2,\,1.9)$.

   d. (5 pts) Find Δz = the change in *z* from $f(1,2)=16$ to $f(1.2,\,1.9)$

   e. (5 pts) Find the differential approximation dz to approximate Δz from part d, above. You may calculate this, directly, or just use previous work and a subtraction.

   f. (5 pts) What is the gradient $\nabla f(1,\,2,16)$?

   g. (5 pts) Find the directional derivative for f, $D_{\bar{u}}$ in the direction of $\bar{u}=\langle 3,-4\rangle$ at the point $(1,\,2,16)$

5. Find the shortest distance between the plane $3x-4y+12z=24$ and the point $P(13,-6,39)$ in three ways:

   a. (10 pts) Use 1st- and/or 2nd- derivative test.

   b. (10 pts) Use earlier skills from Chapter 12.

   c. (5 pts) Use Lagrange Multipliers.

Bonus: Answer up to 3 of the following for up to 15 bonus points.

1. (5 pts) Find the first partials f_x and f_y for $f(x,y)=\int_0^{\sin(x)-5x}\left(\frac{y^2\sinh(\tau)\cos(\tau)}{\tau^2+\pi}\right)d\tau$

2. Answer BOTH if you answer one. Find parametric equations and a vector equation for the line of intersection between the two planes $\begin{array}{l} P_1: \ x+2y-4z=7 \\ P_2: \ 2x+3y+2z=11 \end{array}$ in two ways:

   a. (5 pts) By solving the system using elimination. Matrix method preferred.
   b. (5 pts) By being clever about the direction vector, like Dylan is.

3. (5 pts) (Line segment) Write the equation of the line segment between $A(2,-3,7)$ and $B(-3,2,1)$. What do you obtain if you remove the restriction on t ?

4. (5 pts) Consider the object $9x^2+4z^2-25y=0$. Show its traces in the planes $x=k, y=k, z=k$ for different choices of k and project those into the $yz-, xz-,$ and $xy-$planes, respectively.